Mathematics in the Age of Computation

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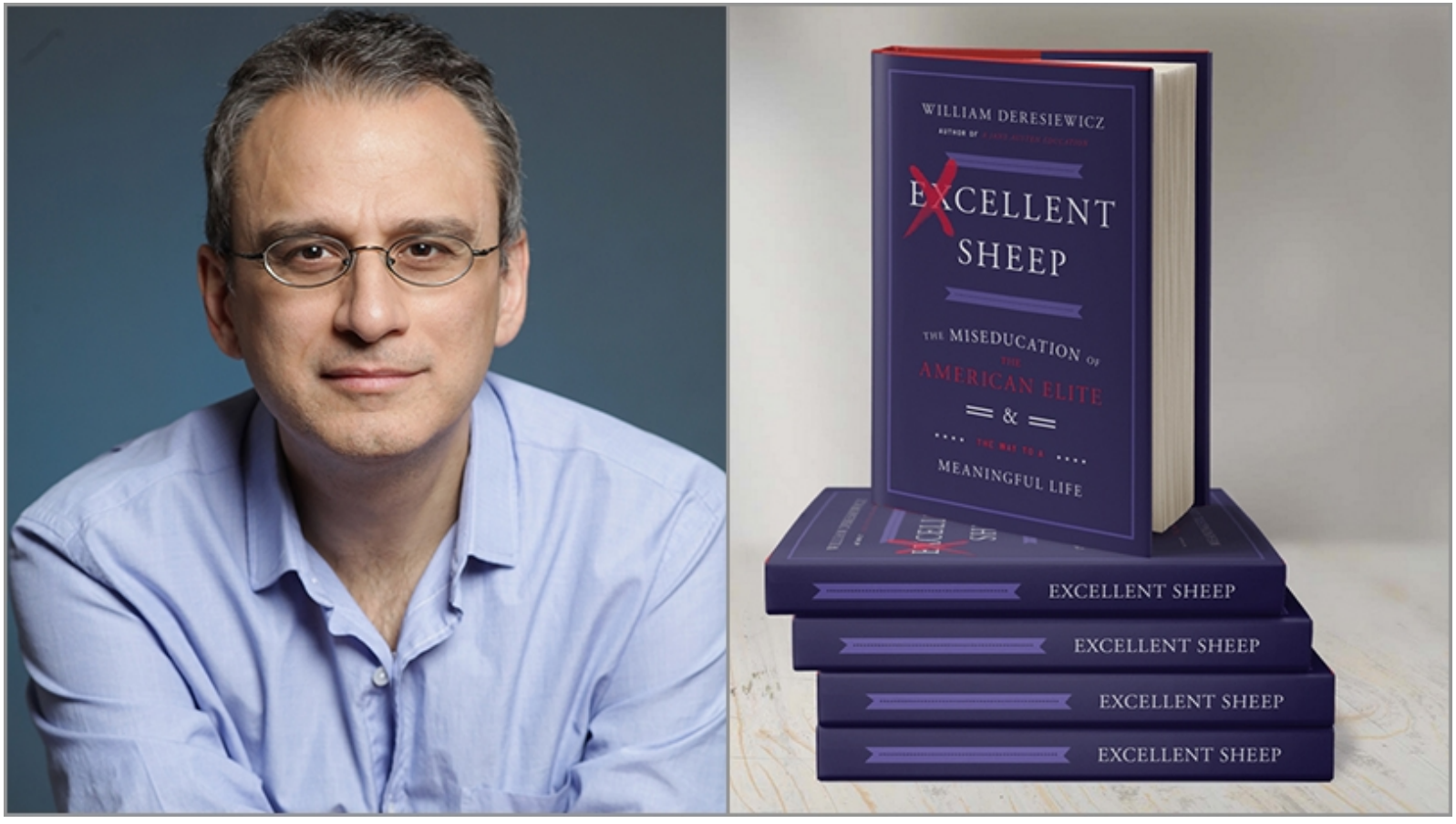
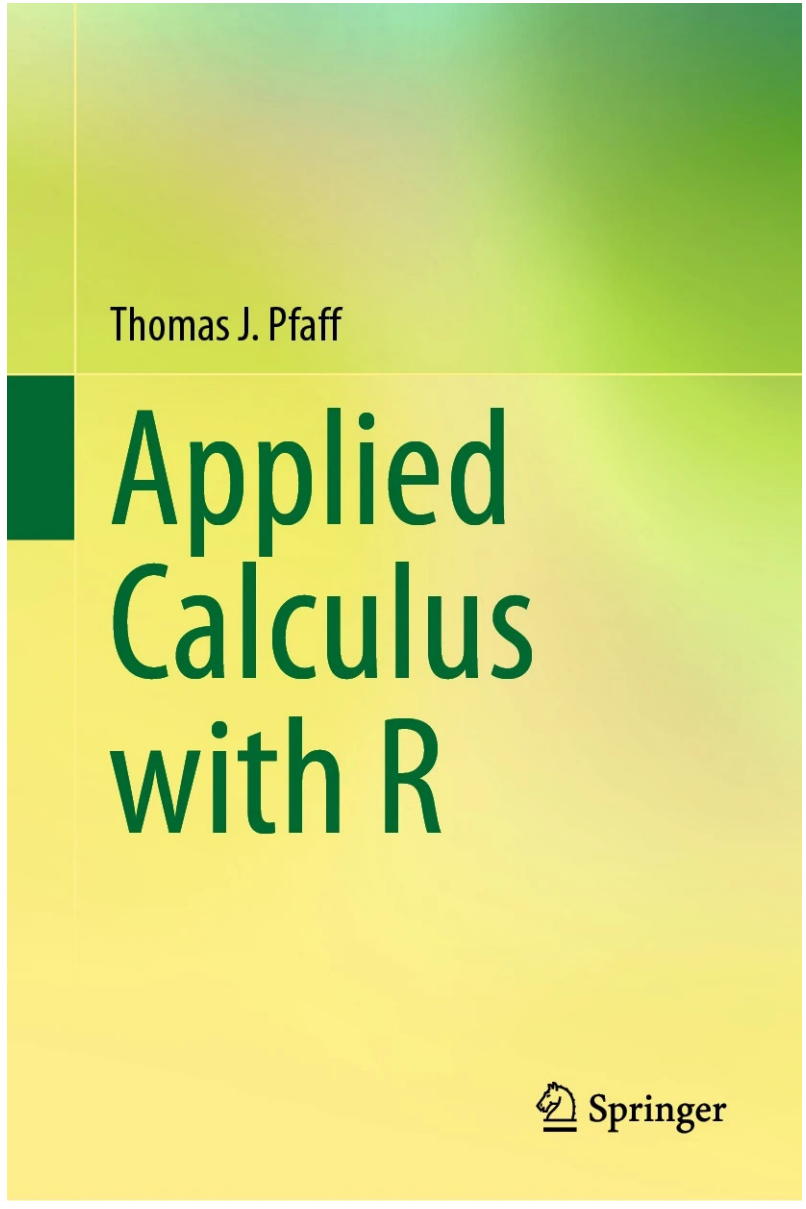
Oct 20, 2023

Challenge!

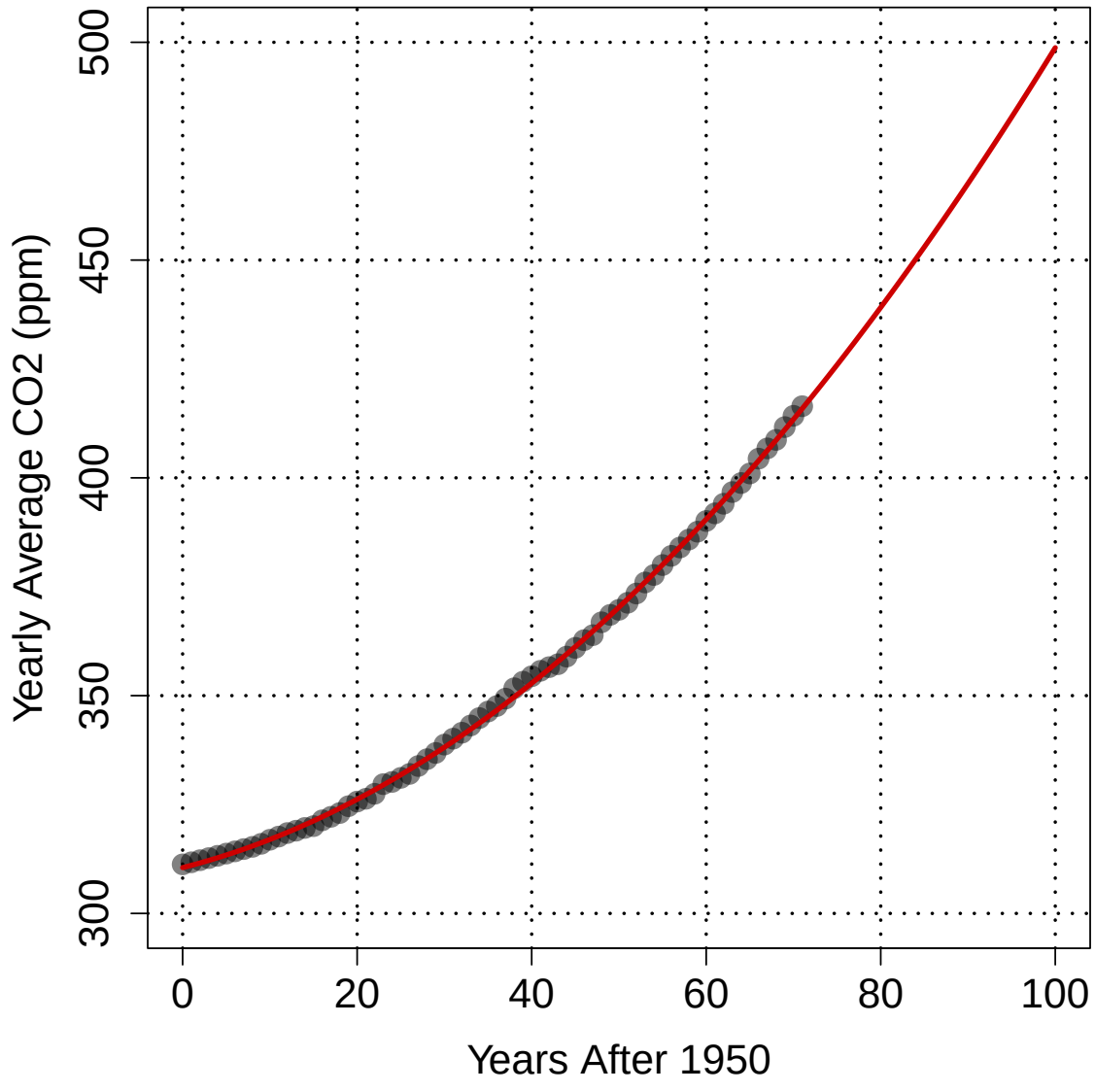
Keep me from getting to the last slide!

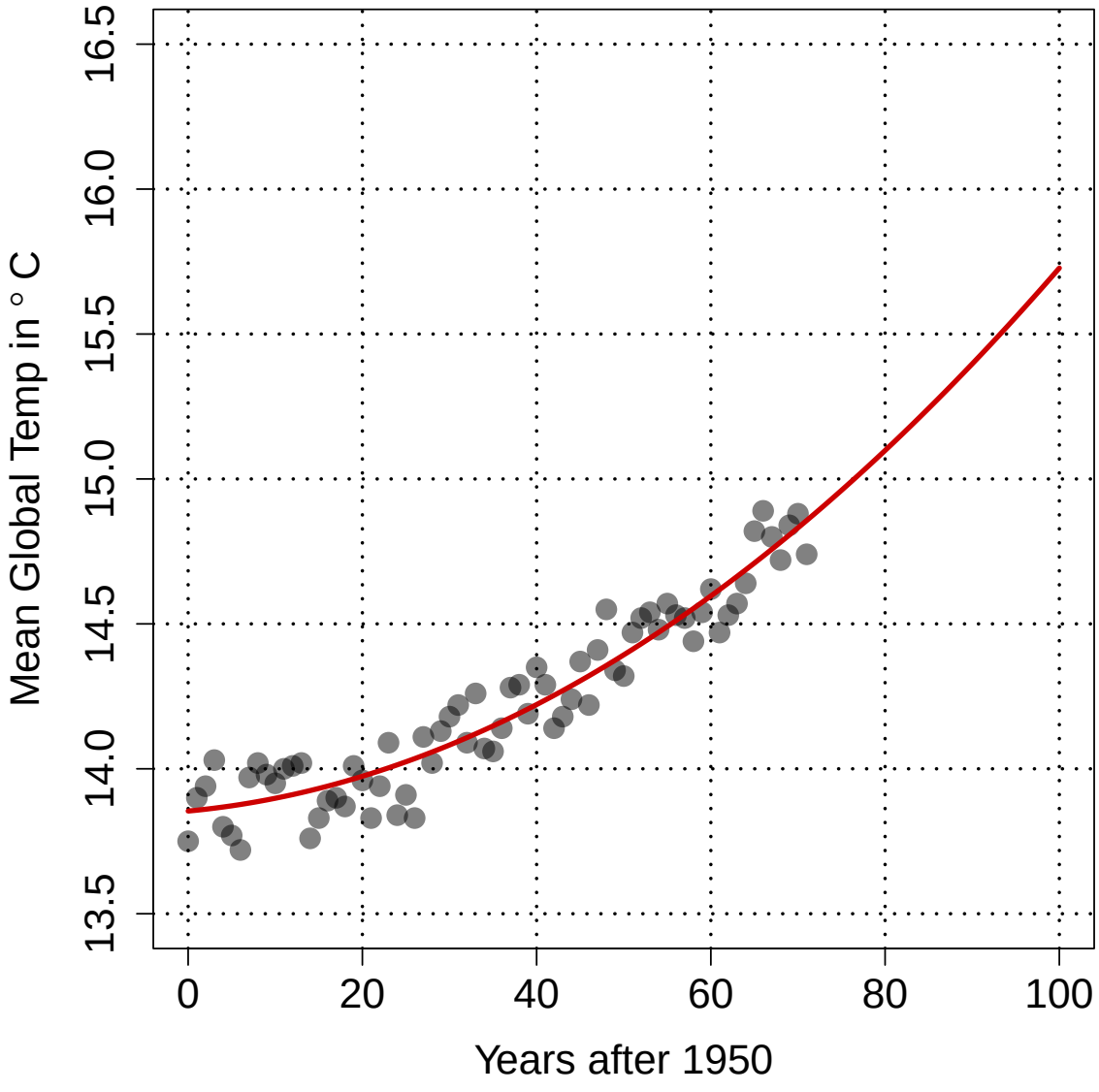


William Deresiewicz



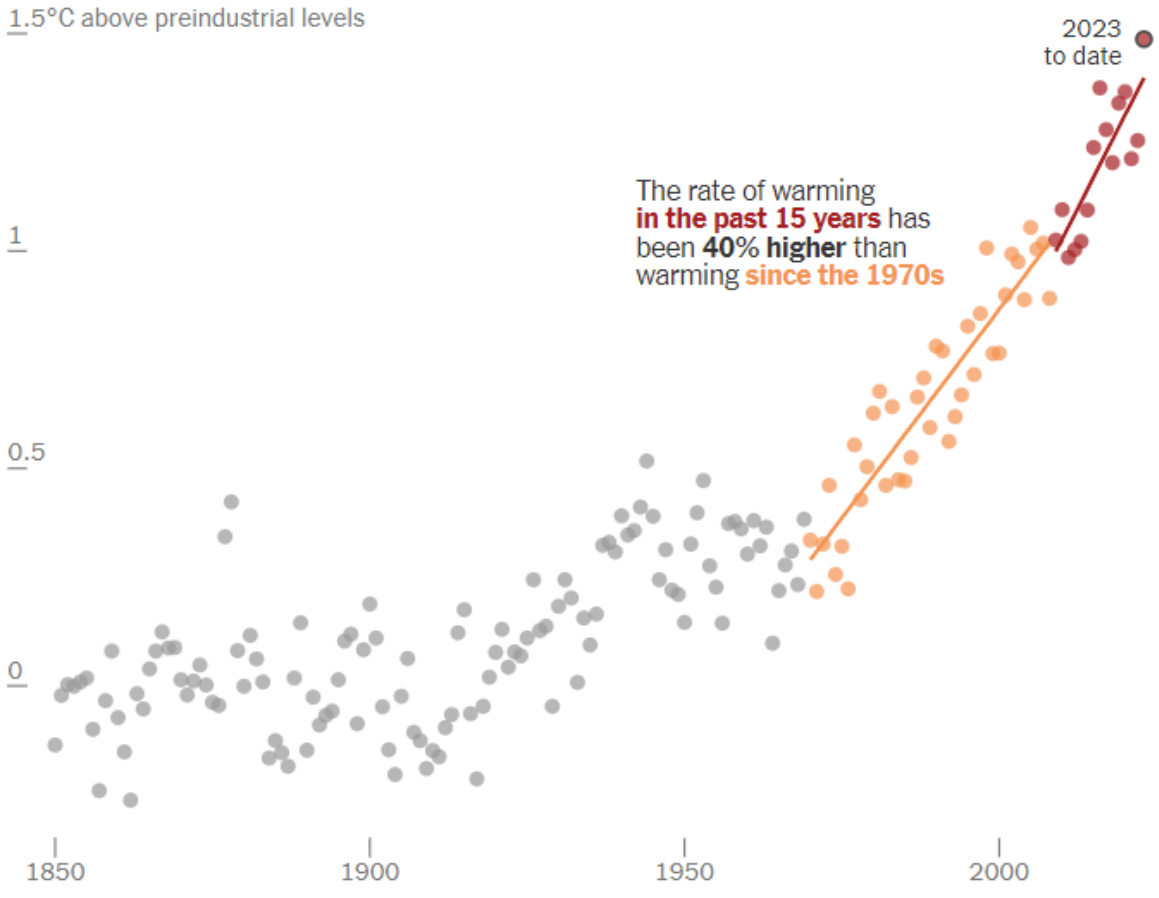
(Shameless Self Promotion)


Yearly Average CO2 (ppm)
300
350
400
450
500
0
20
40
60
80
100
Years After 1950

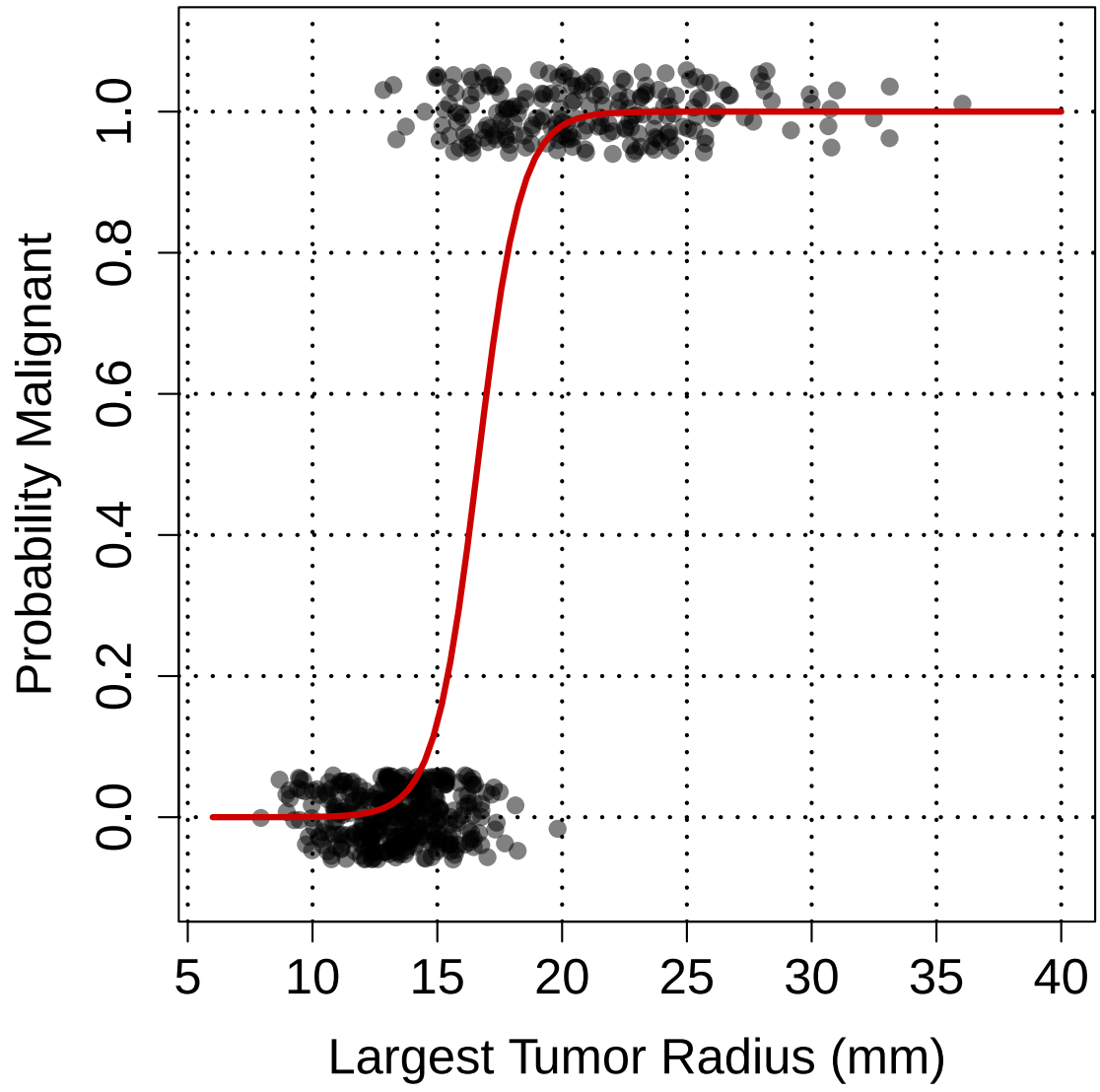

Mean Global Temp in ° C
13.5
14.0
14.5
15.0
15.5
16.0
16.5
0
20
40
60
80
100
Years after 1950

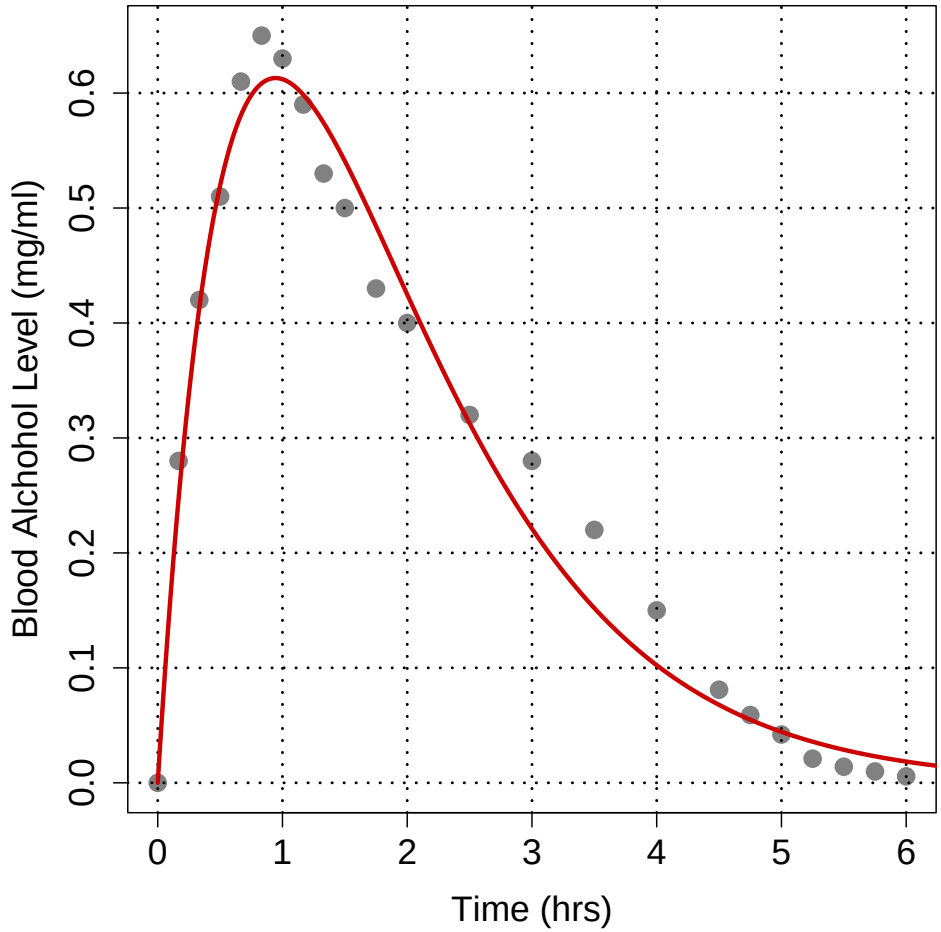
Global warming may have accelerated in the past 15 years

Annual average temperatures since 1850

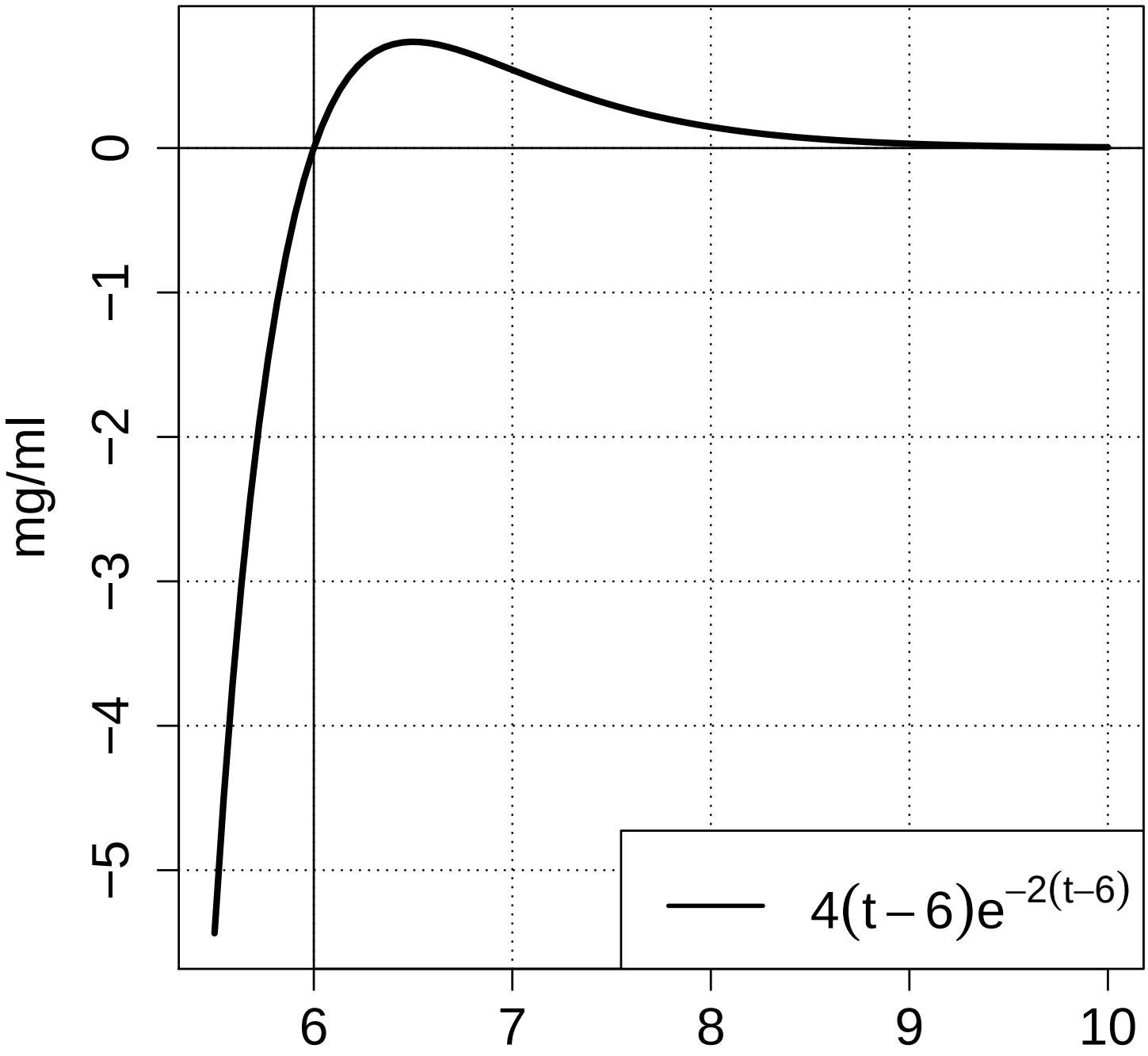


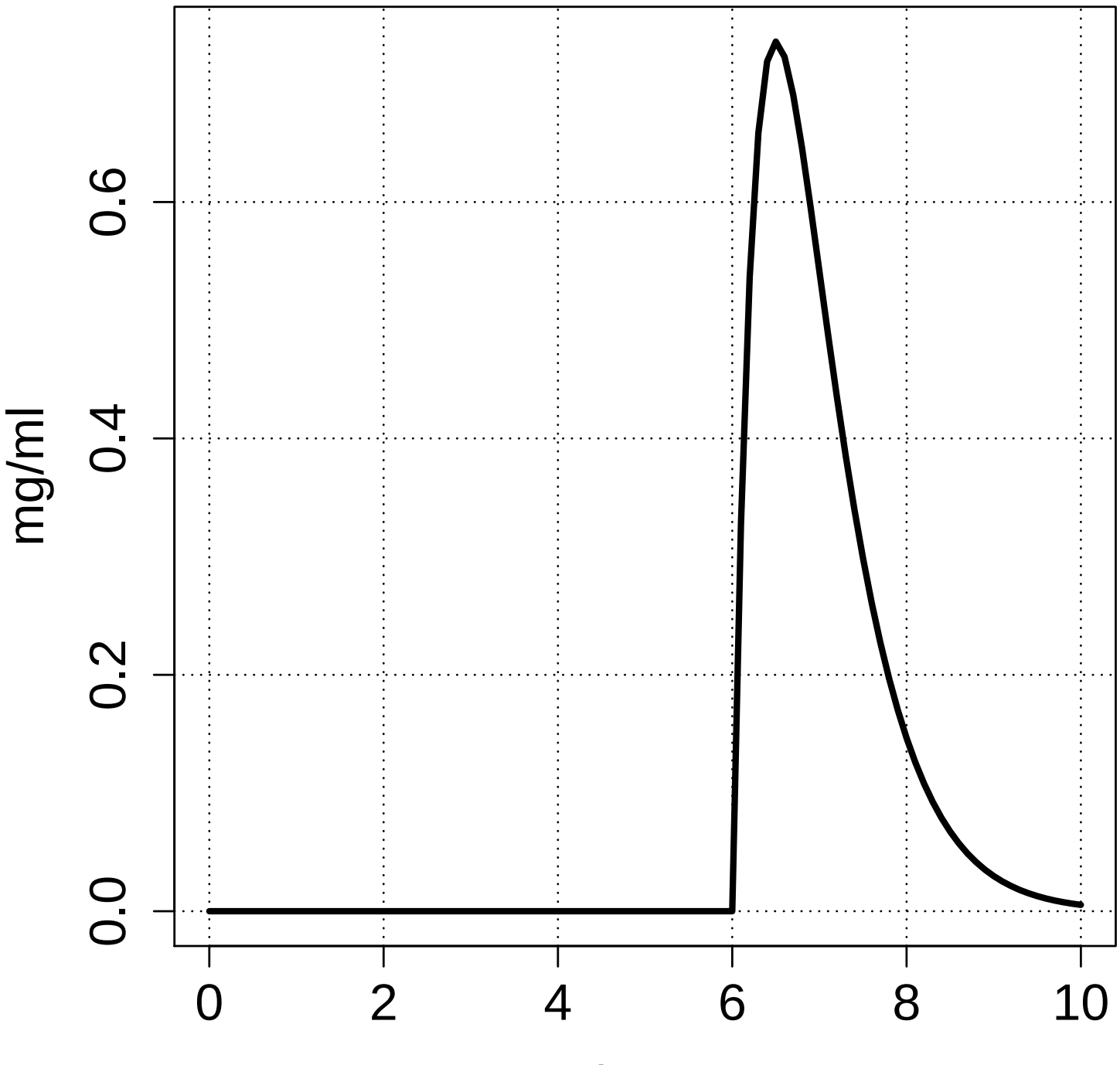
Source: Berkeley Earth Land/Ocean Temperature Record


Probability Malignant
0.0
0.2
0.4
0.6
0.8
1.0
5
10
15
20
25
30
35
40
Largest Tumor Radius (mm)

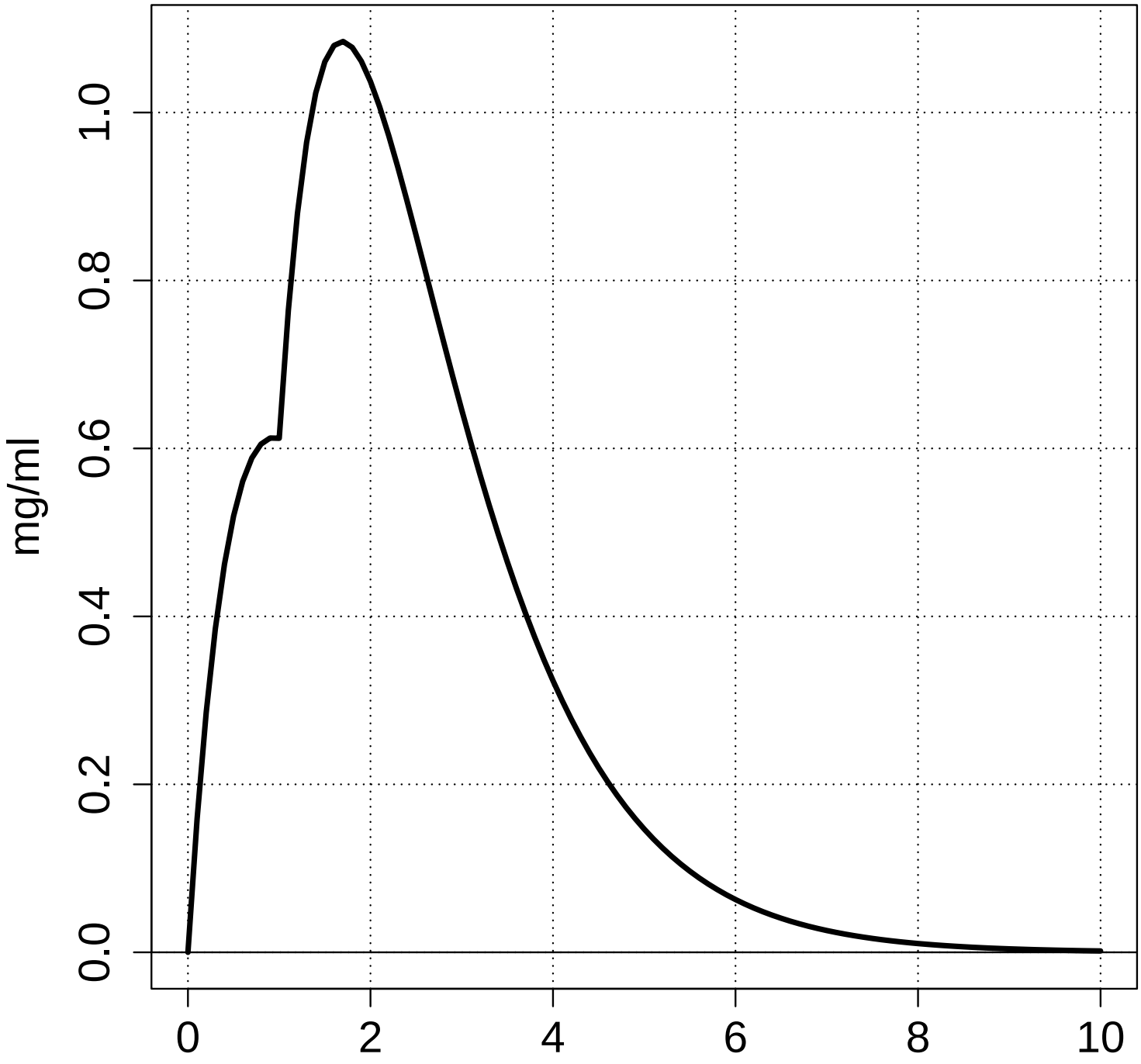


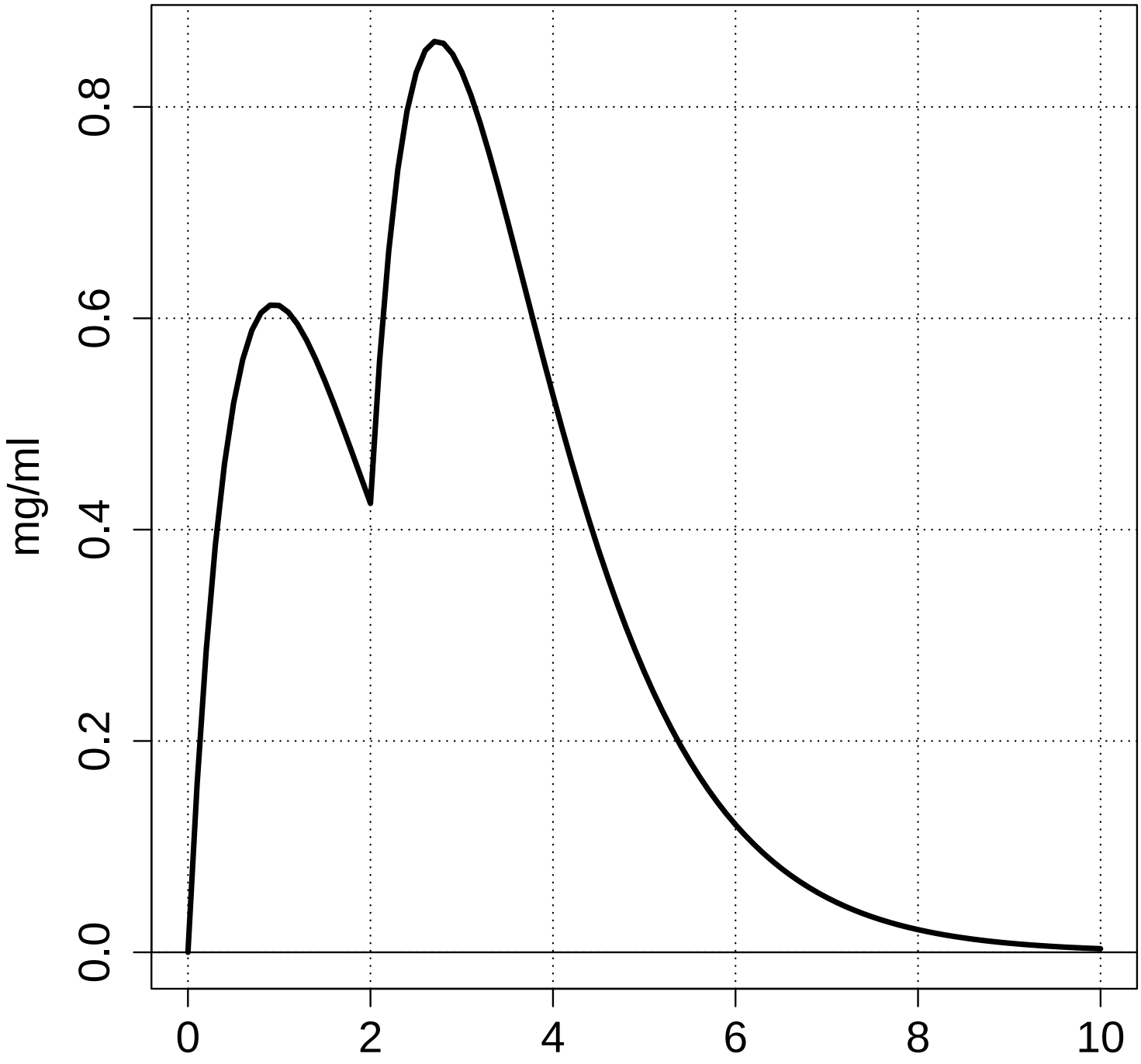
Blood alcohol concentration of eight fasting adult males after consuming a 95% ethanol oral dose of 45ml. The surge function is given by $S(x) = 1.7639364204620 5xe^{-1.05841662684339x}$. Note that driving impairment begins around 0.5 mg/ml and the legal limit in most states is 0.8mg/ml. The dosage here is about 2.5 standard shots of vodka or about one and a half 16oz of 6% beer.


mg/ml
0
−1
−2
−3
−4
−5
6
7
8
9
10
$4(t-6)e^{-2(t-6)}$

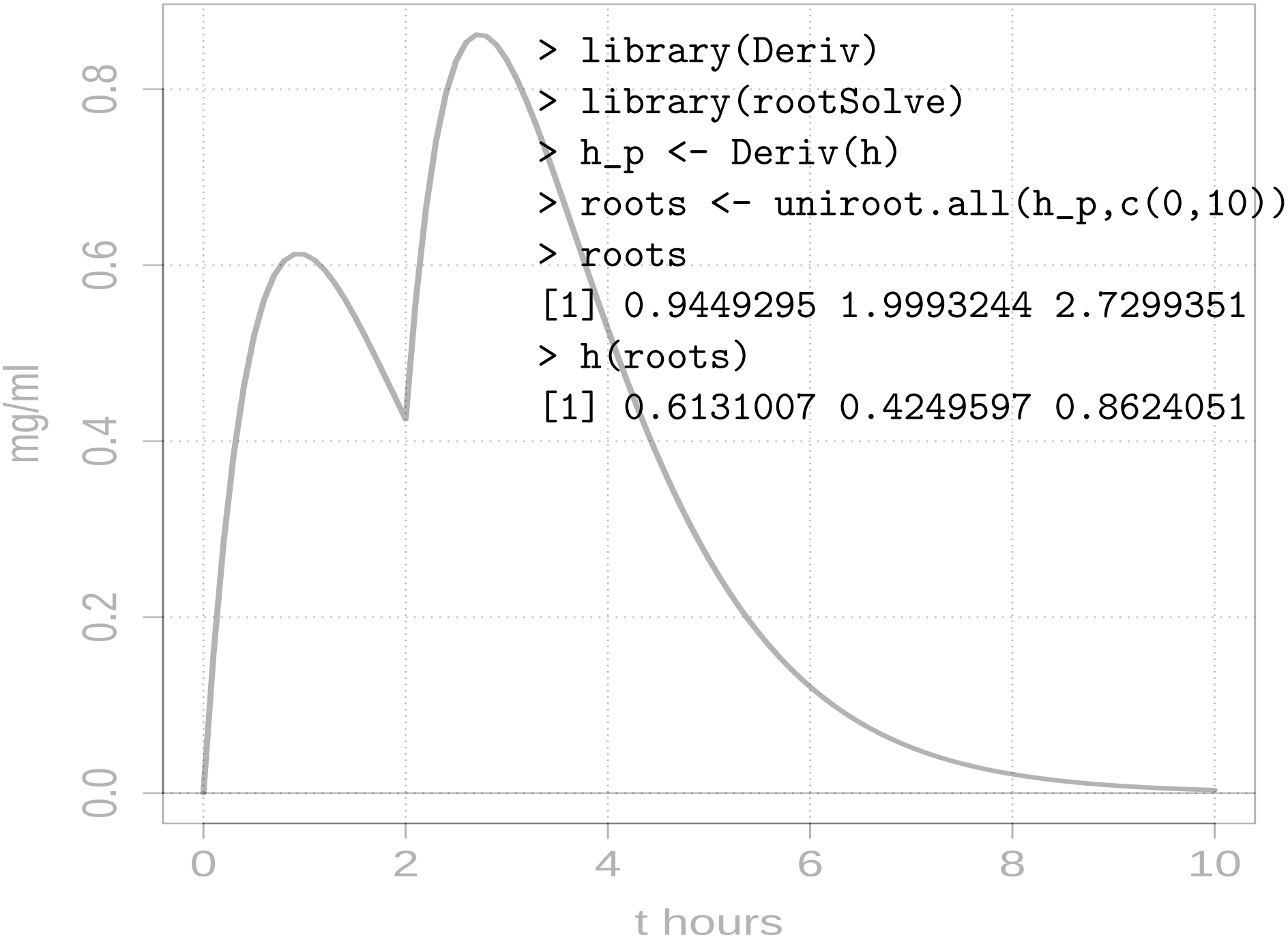

mg/ml
0.0
0.2
0.4
0.6
0
2
4
6
8
10

$$f(t) = \begin{cases} 0 & t \leq 6 \\ 4(t-6)e^{-2(t-6)} & t > 6 \end{cases}$$


mg/ml
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10


mg/ml
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
10

```
par(mar=c(5,5,2,2))
Surge <- function(x)
  {1.76393642046205*x*exp(-1.0584166268433 9*x)}
Surge_Piece <- function(s,x){ ifelse(x<s,0,Surge(x-s))}
h <- function(x){Surge(x)+Surge_Piece(2,x)}
curve(h,0,10,xlab="t hours", ylab="mg/ml",
  lwd=3,cex.axis=1.5,cex.lab=1.5)
abline(h=0)
grid(NULL,NULL,col="black")
```


> library(Deriv)
> library(rootSolve)
> h_p <- Deriv(h)
> roots <- uniroot.all(h_p,c(0,10))
> roots
[1] 0.9449295 1.9993244 2.7299351
> h(roots)
[1] 0.6131007 0.4249597 0.8624051
mg/ml
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
10
t hours

New Model

A population grows proportional to its size.

New Model

A population grows proportional to its size.

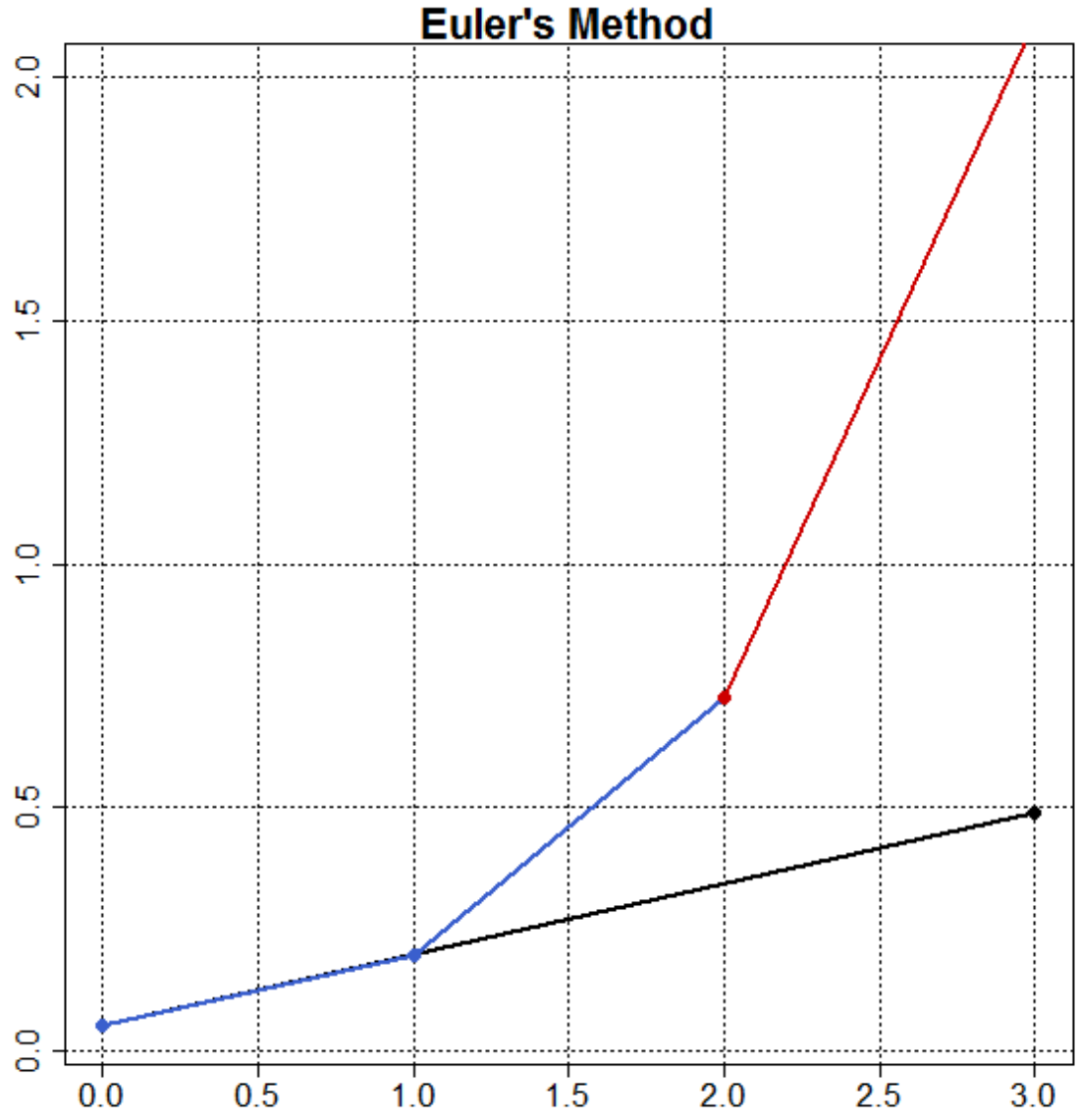
$$P'(t) = rP(t)$$

Building an SIR Model

$$S(t) = \text{Number susceptible at time } t$$
$$I(t) = \text{Number infected at time } t$$
$$R(t) = \text{Number recovered at time } t$$

An SIR Model

$$
\begin{aligned}
S'(t) &= -mcI(t)S(t) \\
I'(t) &= mcI(t)S(t) - \frac{I(t)}{d} \\
R'(t) &= \frac{I(t)}{d}
\end{aligned}
$$



$$\frac{dy}{dx} = 3\left(1 - \frac{y}{2}\right)y$$

$y(0) = 0.05$

3 Steps

$x = 0$ to $x = 3$

$\Delta x = 1$

$y'(2) \approx 1.38839$

$y(3) \approx y'(2)1 + y(2) = 2.11562$

Frame 3

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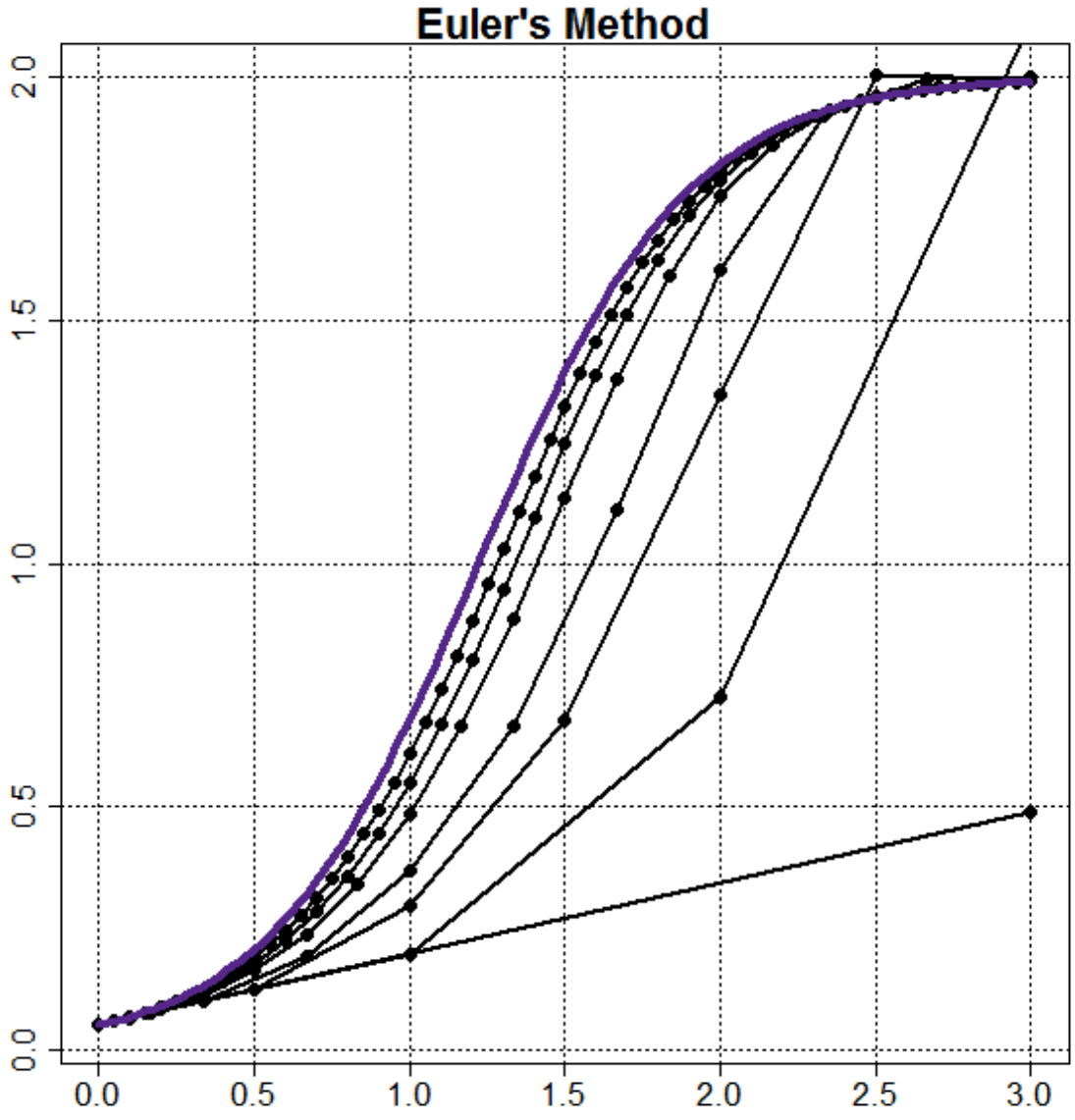
$$y = m(x - x_1) + y_1$$

Historical Note

The Euler method is named after Leonhard Euler, who first proposed it in his book Institutionum calculi integralis (published 1768–1770).



$$\frac{dy}{dx} = 3\left(1 - \frac{y}{2}\right)y$$

$$y(0) = 0.05$$

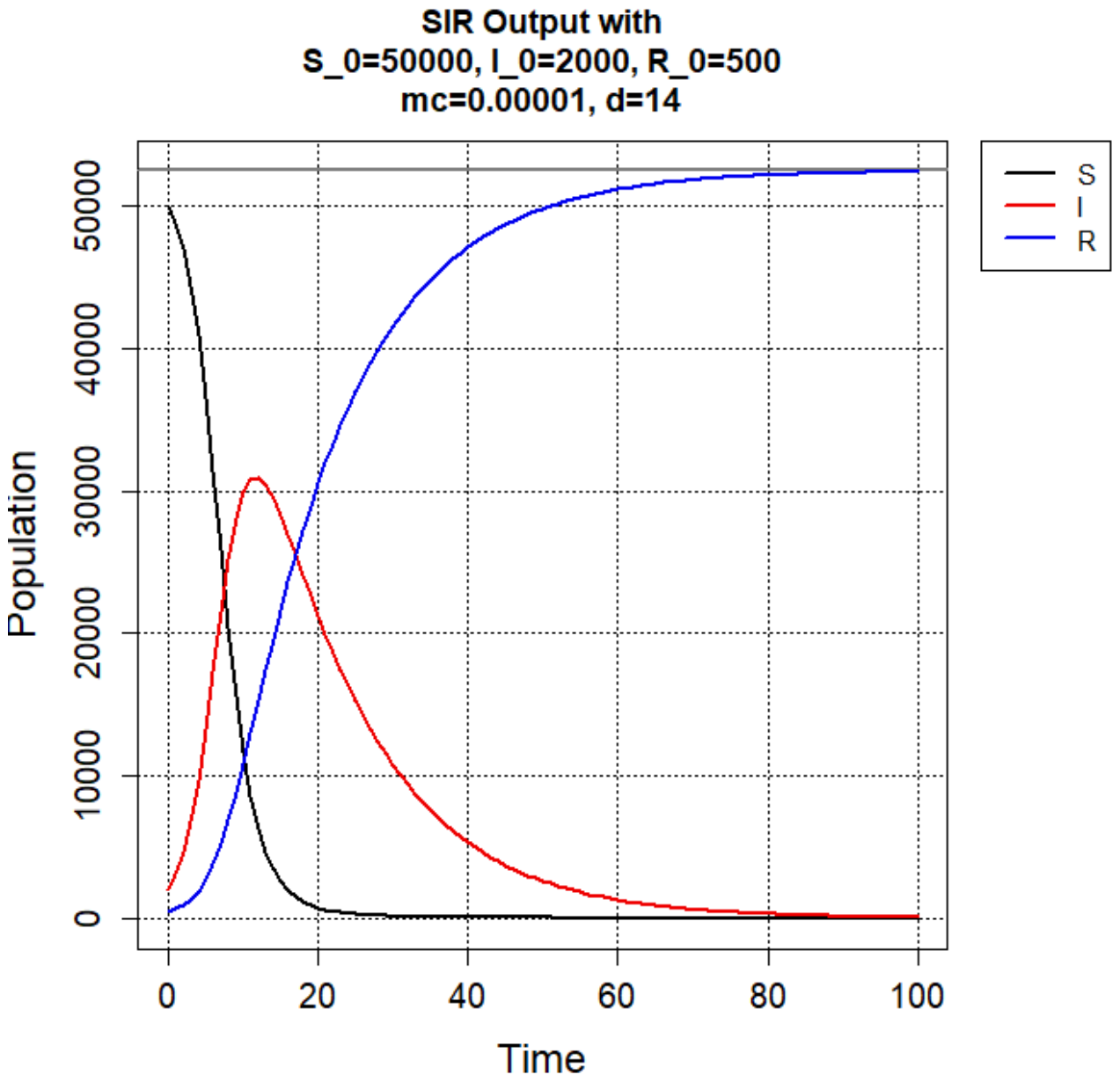
60 Steps

$x = 0$ to $x = 3$

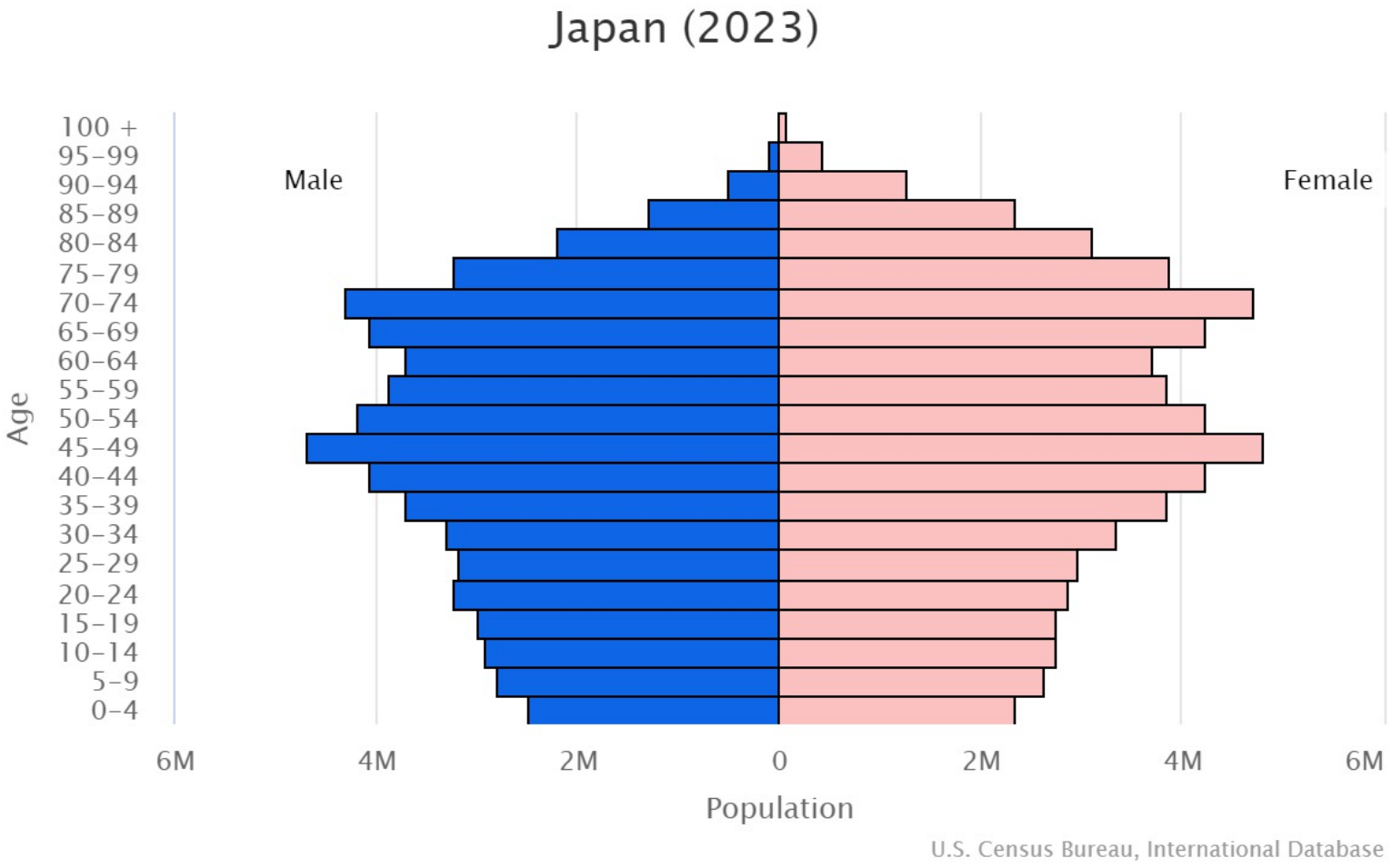
$\Delta x = 0.05$

Solution

SIR Example


SIR Output with
S_0=50000, I_0=2000, R_0=500
mc=0.00001, d=14
Population
0
10000
20000
30000
40000
50000
0
20
40
60
80
100
Time
S
I
R

SIR Type Scenario



Thank You!

Questions and Conversation?
https://sustainabilitymath.org/
https://briefedbydata.substack.com/