The UPDATED Function Gallery

Each graph below is a real world data set with a fitted curve. The fitted curves will be used throughout the text. Note that each graph caption provides the function of the fitted curve, as well as, the beginning and end of the data set where appropriate. These functions are here to generally demonstrate the idea of curve fitting as a way functions are created and specifically to see how well the curves fit the particular data. Examples that are current will be updated and posted on the companion web site https://sustainabilitymath.org/acr/ along with the data and R code for the curve fitting. The exercises at the end of this chapter will help you get used to using functions with units and context. **TIP:** Once you type these function into an R script you should save the code so you don't have to type the function into R repeatedly. **WARNING:** If you copy and paste the functions in the captions from a pdf into R it may not work, especially if you are using a mac. The most common problems is the negative sign is incorrect, but it can be deleted and retyped.

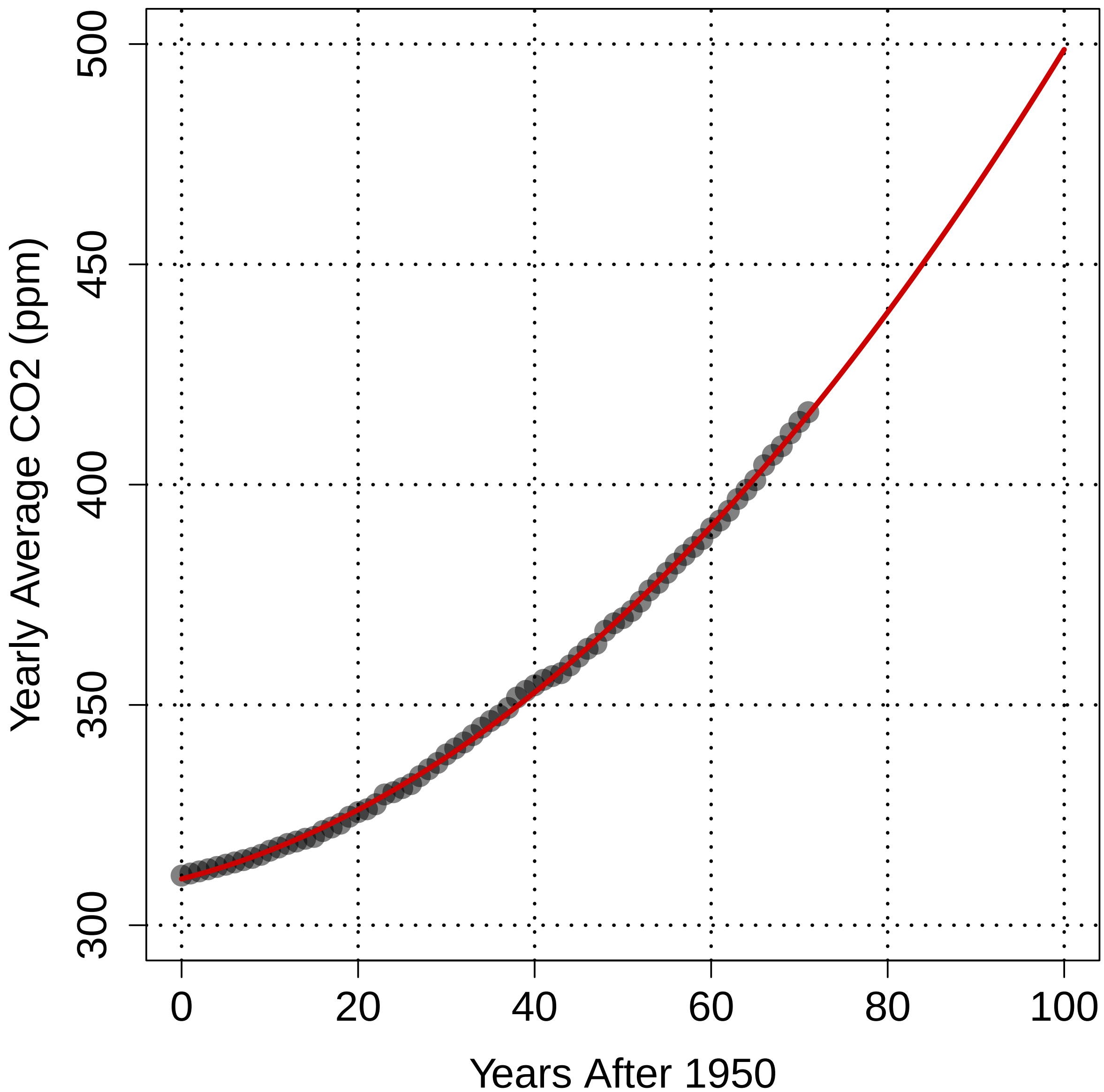

Figure 1: $CO2(t) = 0.0137331303009963t^2 + 0.5091468932687 88t + 310.512512186844$ average yearly atmospheric CO2 at Mauna Loa in ppm t years after 1950. The data is from 1950 through 2021. [8]

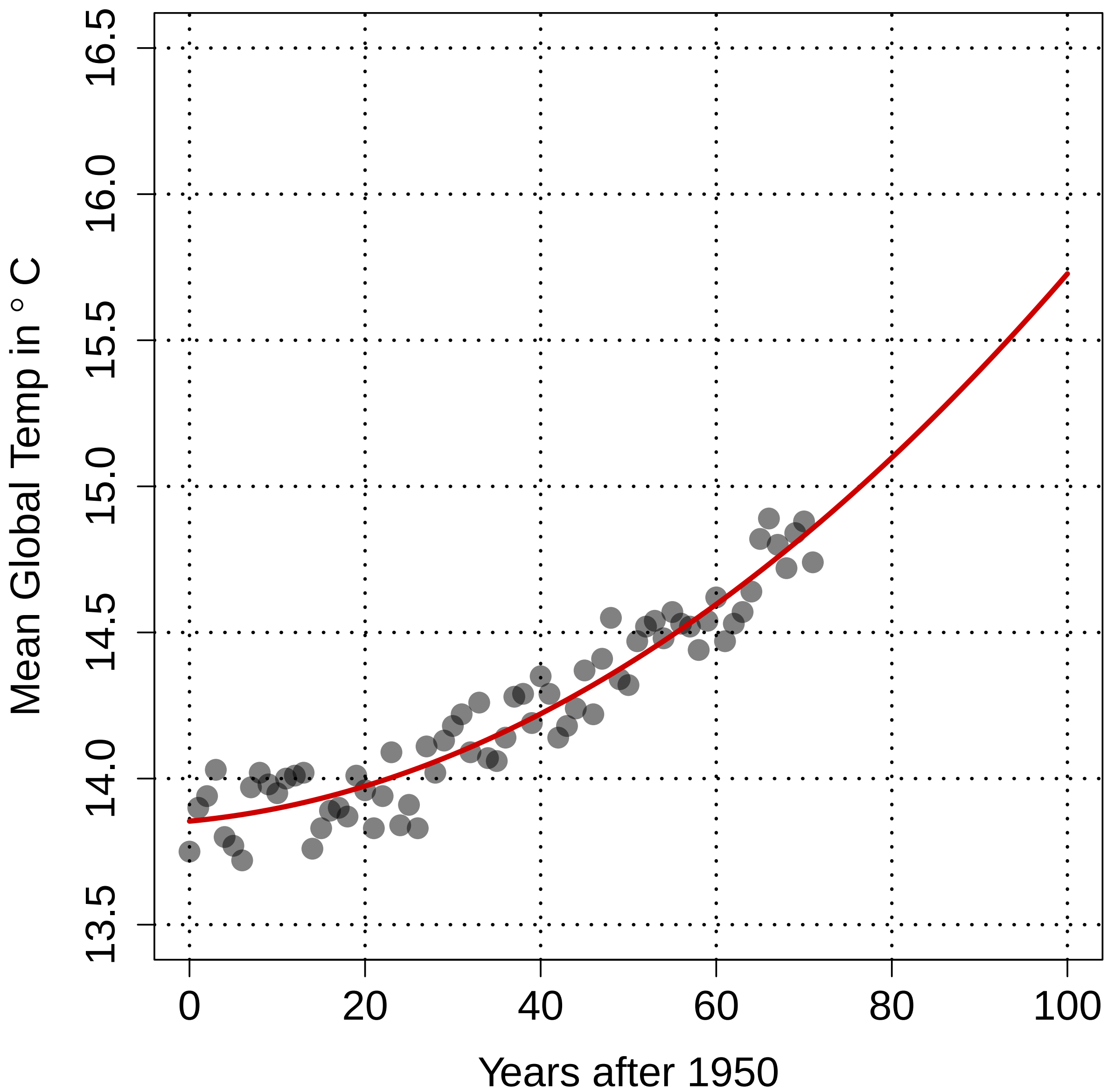

Figure 2: $GTemp(t) = 0.000159118973994531t^2 + 0.00282082411785264t + 13.8542160310996$ global average temperature in degrees Celsius t years after 1950. The data is from 1950 through 2021. [6]

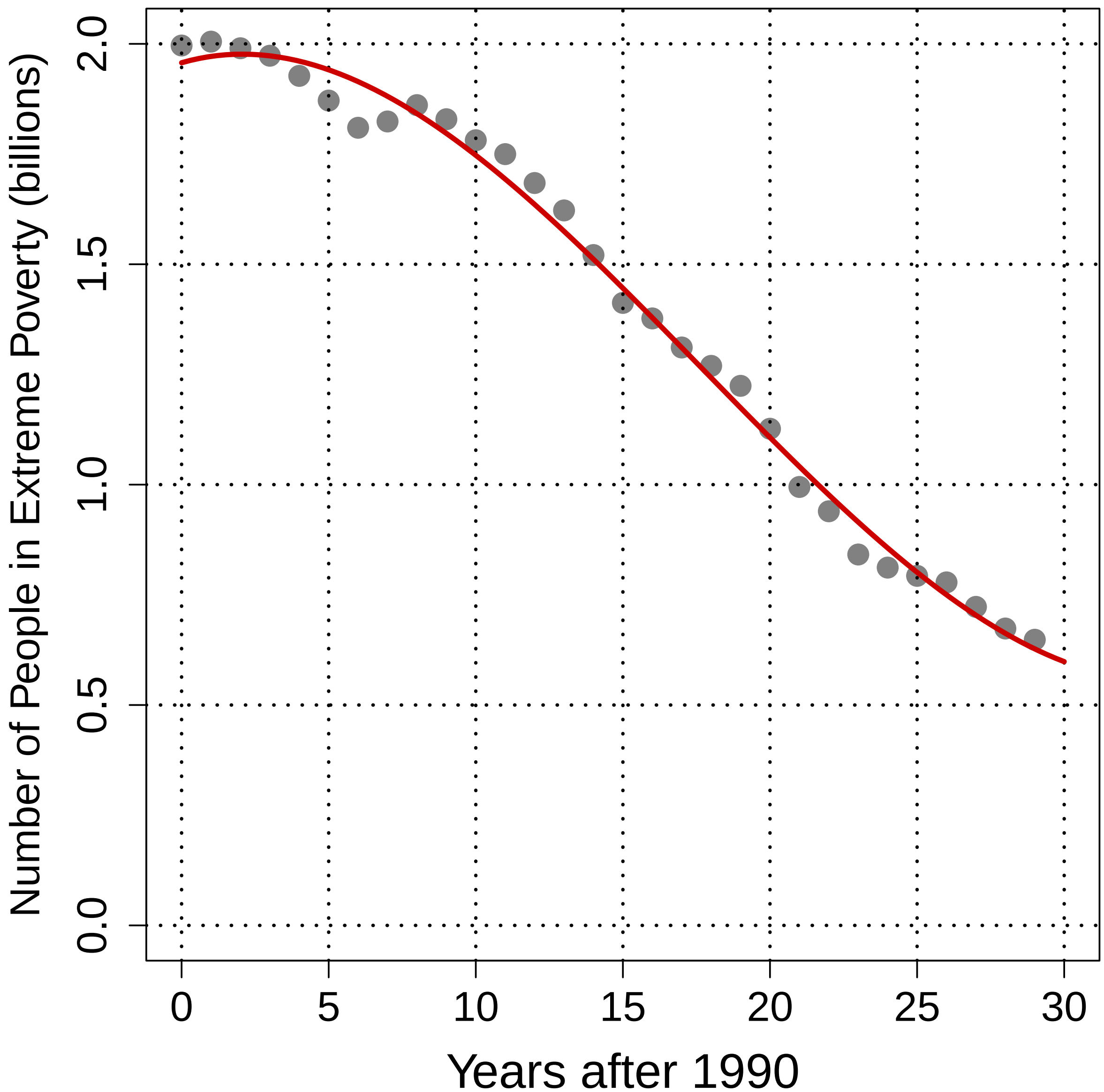

Figure 3: $P(t) = 1.95718412183052 + 0.0192626524183241t + -0.00496217402961009t^2 + 0.0000936768151746741t^3$ number of people in extreme poverty in billions, t years after 1990. Extreme poverty is defined as living below the International Poverty Line of $2.15 per day. This data is adjusted for inflation and for differences in the cost of living between countries. The data is from 1990 through 2019. Note that overall world population has been growing. [3]

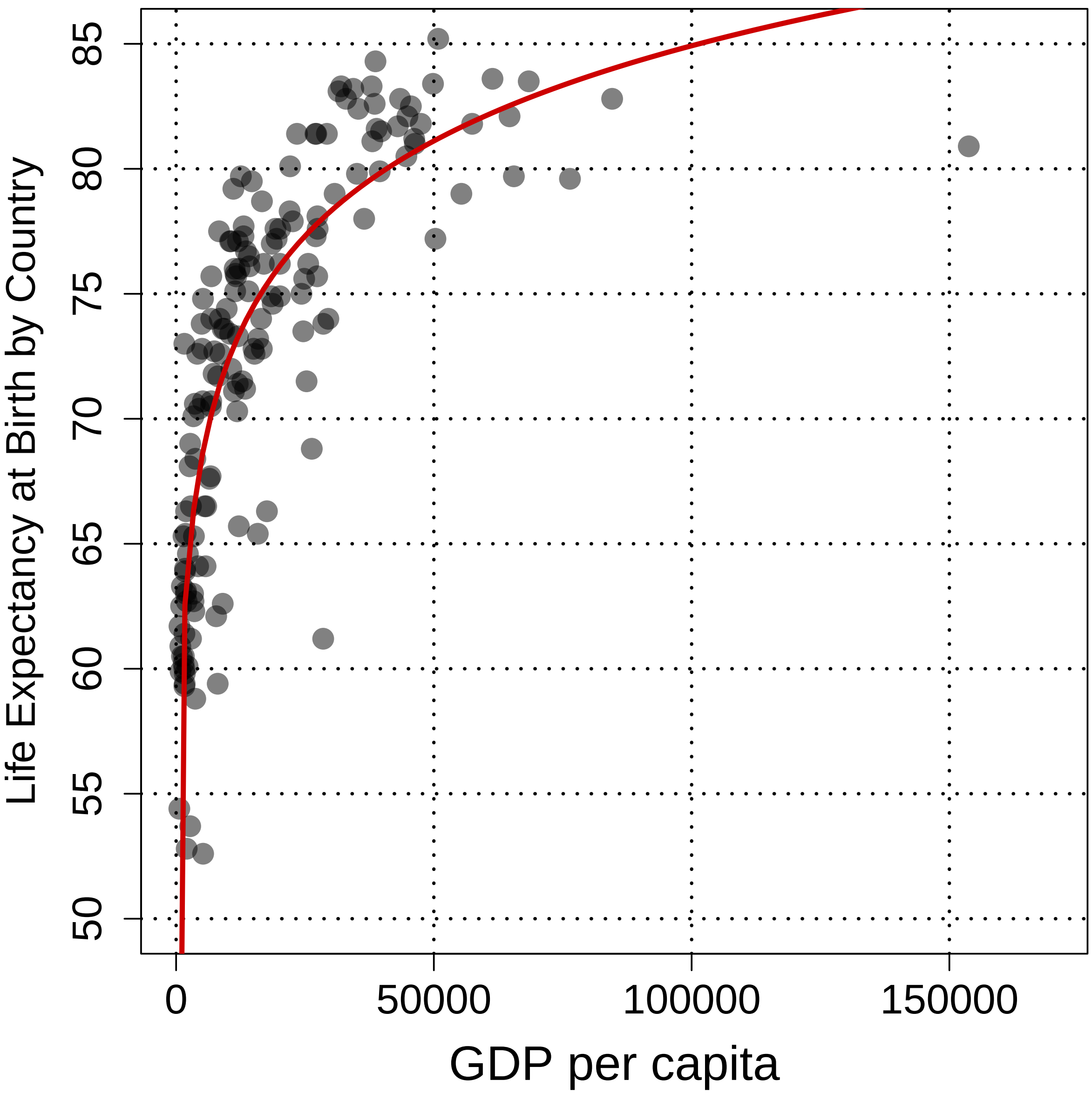

Figure 4: $LeGdp(x) = 5.49220288353388 log(x + 1) + 21.6908132339644$ life expectancy at birth for 166 countries in 2018, where x is GDP per capita in 2011 international dollars, which corrects for inflation and cross-country price differences. The outlier is Qatar. Note $log(x + 1)$ is the natural log. [7]

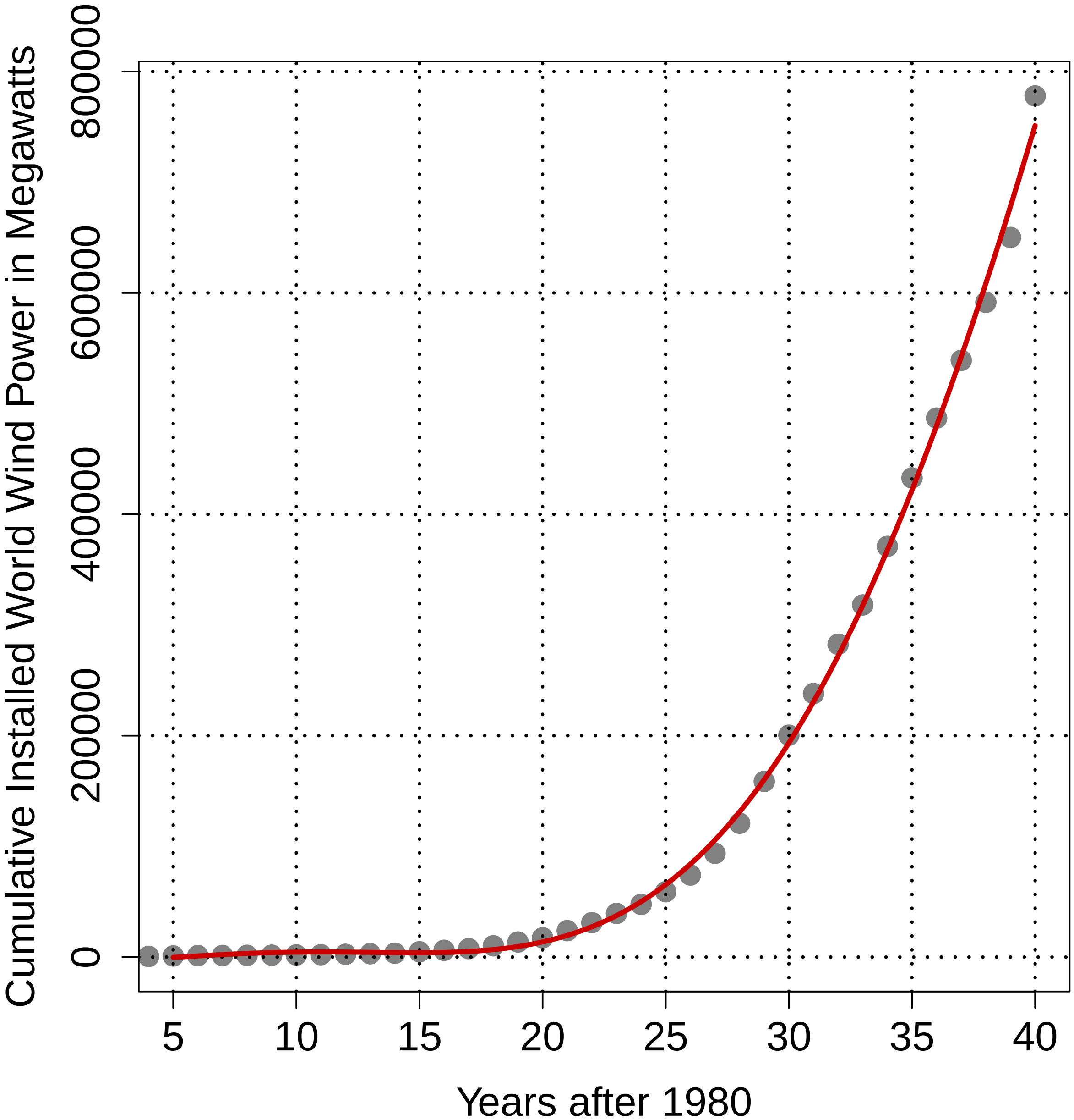

Figure 5: $Wwind(t) = -0.034842402673882t^5 + 3.66966512288051t^4 - 105.918177599909t^3 + 1169.39739731949t^2 - 4260.57451226362t + 2859.79180744146$ cumulative installed world wind power in megawatts t years after 1980. The data is from 1980 through 2020. [4]

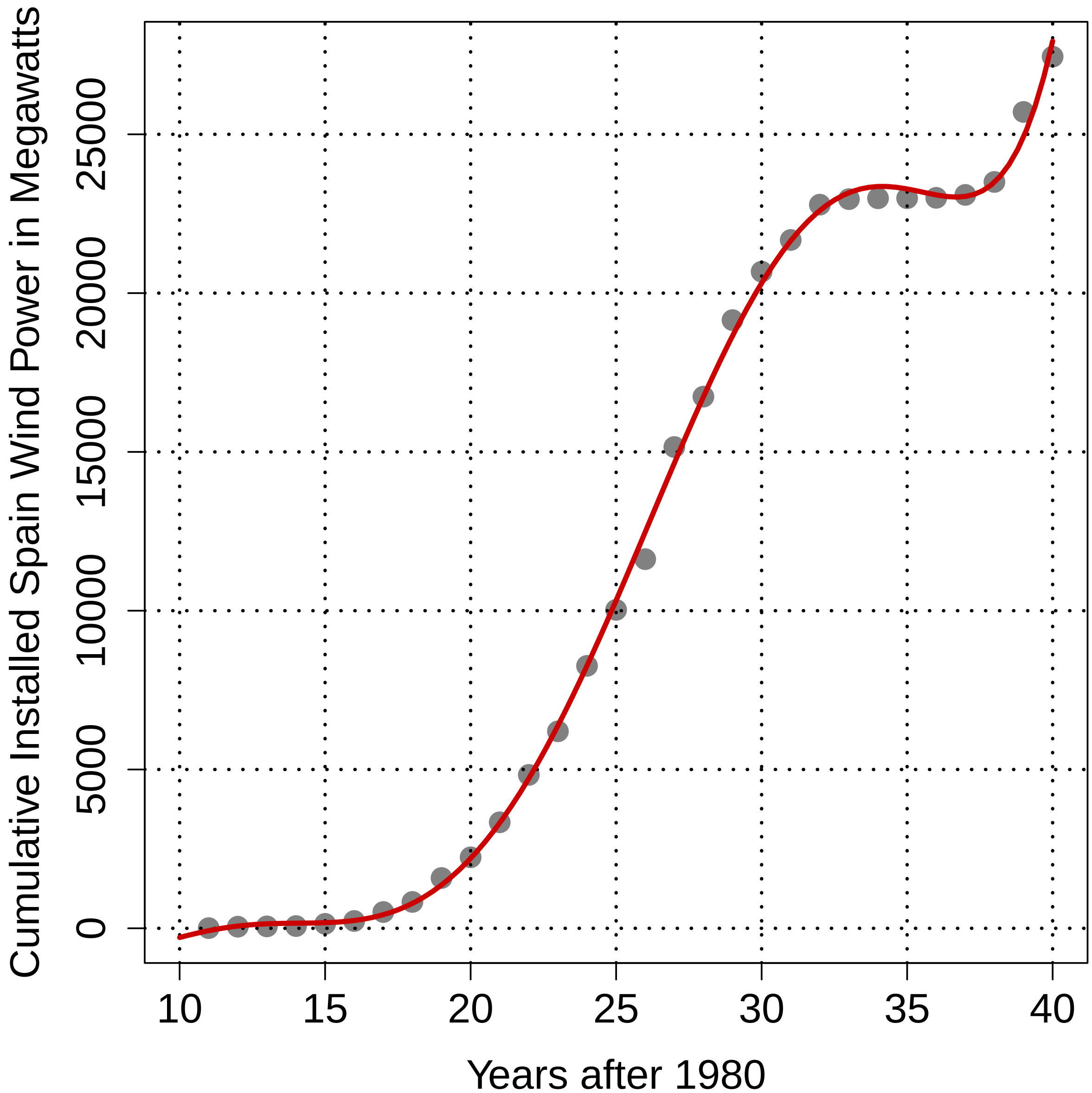

Figure 6: $Swind(t) = 0.00161675544440024t^6 + -0.207341202671209t^5 + 10.24714368839421t^4 + -247.214861506442t^3 + 3087.43087123947t^2 + -18940.4936525017t + 44234.0826986463$ cumulative installed Spain wind power in megawatts t years after 1980. The data is from 1992 through 2020. [4]

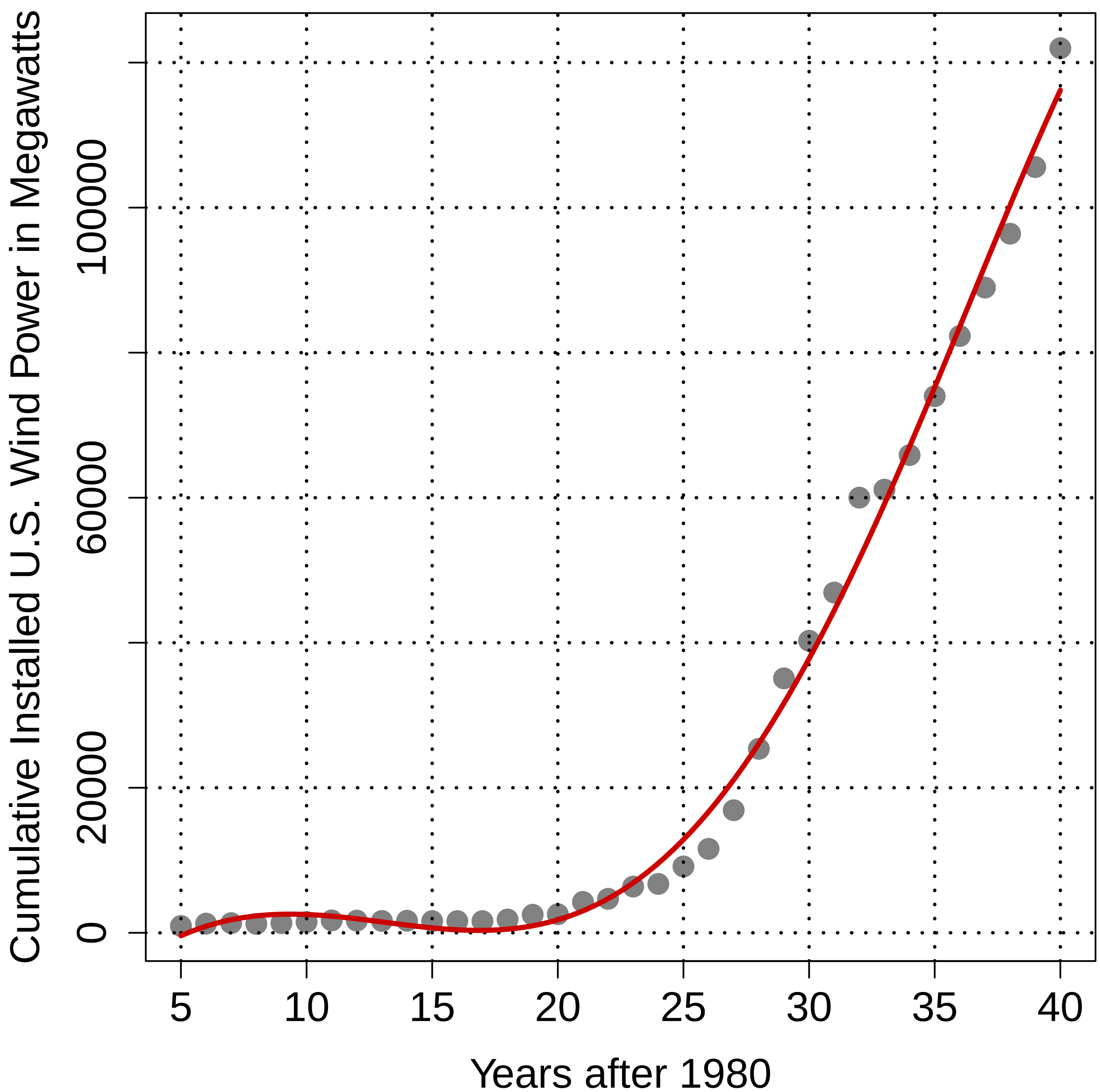

Figure 7: $USwind(t) = -0.00606729846765557t^5 + 0.403796445970404t^4 - 279.748222478446t^2 + 4142.935347737984t - 14361.298551044$ cumulative installed U.S. wind power in megawatts t years after 1980. The data is from 1985 through 2020. [4]

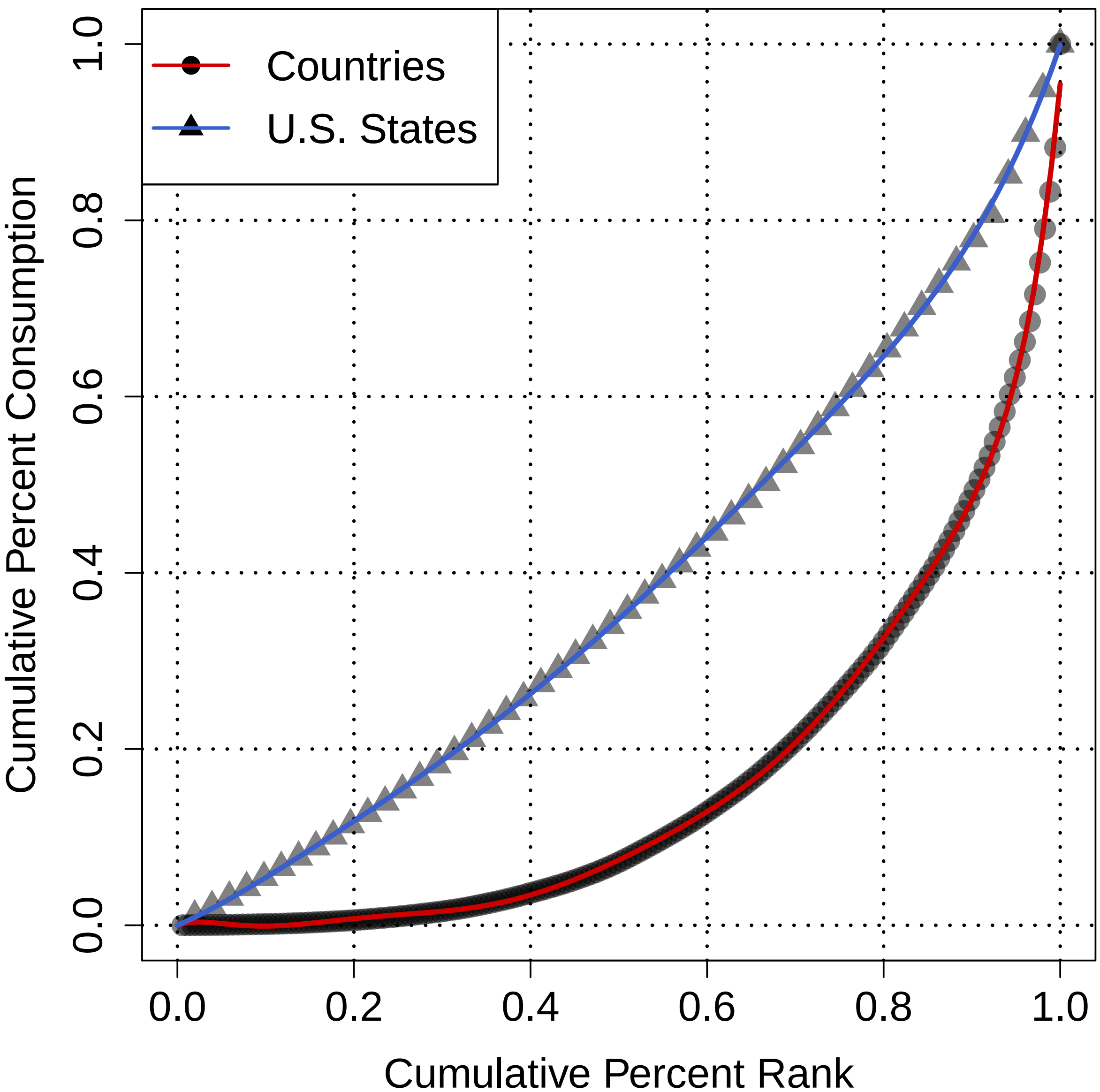

Figure 8: The distribution of energy consumption in the U.S. (2014 data) and World (2011 data) can be modeled by $ECus(x) = 7.2038917391x^6 - 17.8551679663x^5 + 16.5816140612x^4 - 7.0654275059x^3 + 1.7077246274x^2 + 0.4260396828x$ and $ECw(x) = 678.0352163746x^9 - 2796.2519054480x^8 + 4802.0852334478x^7 - 4441.8091503689x^6 + 2389.4054597788x^5 - 751.8800491391x^4 + 132.3874503758x^3 - 11.3747211453x^2 + 0.3569478992x$. To interpret these functions consider the example: $ECus(0.63) = 0.47$ means that the bottom 63% of states in the U.S. have per capita energy use in the bottom 48% of all states. The function ECw(x) is similar and replaces countries for states. [2]

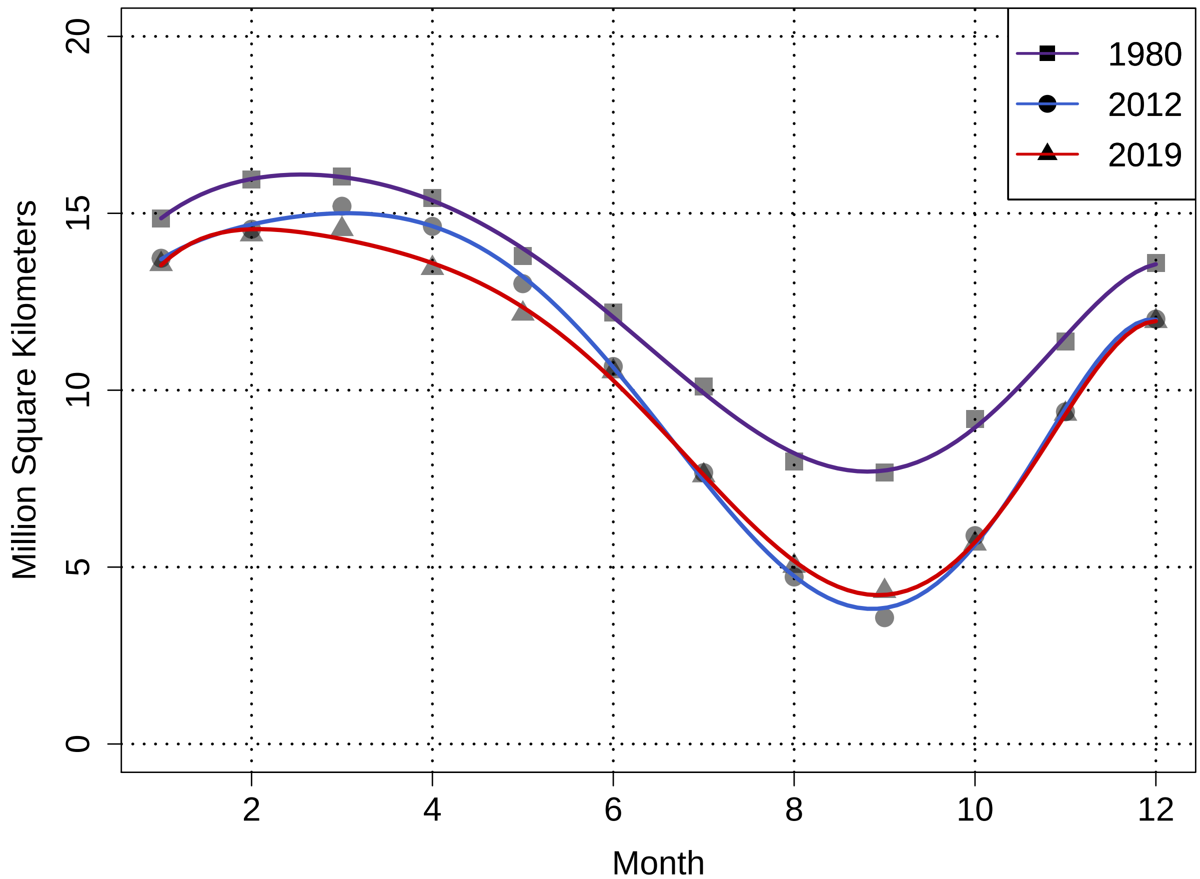

Figure 9: Average monthly Arctic ice extent in million square kilometers with curve fits for 1980, 2012 (current record low year), and 2019. We interpret 1 to mean the middle of January, etc.

$AI\_1980(x) = 11.4612878787871 + 5.3194875879835x - 2.4766110734985x^2 + 0.6539677816055x^3 - 0.1073611634825x^4 + 0.0087432283811x^5 - 0.0002627314815x^6$

$AI\_2012(x) = 10.7484090909081 + 4.9595950483327x - 2.7846102155594x^2 + 0.9400642525363x^3 - 0.1762030228758x^4 + 0.0150792326546x^5 - 0.0004622140523x^6$

$AI\_2019(x) = 7.900757575757 + 9.987580105349x - 5.796657844468x^2 + 1.682711461332x^3 - 0.262366871962x^4 + 0.019775106838x^5 - 0.000558959695x^6$. [5]

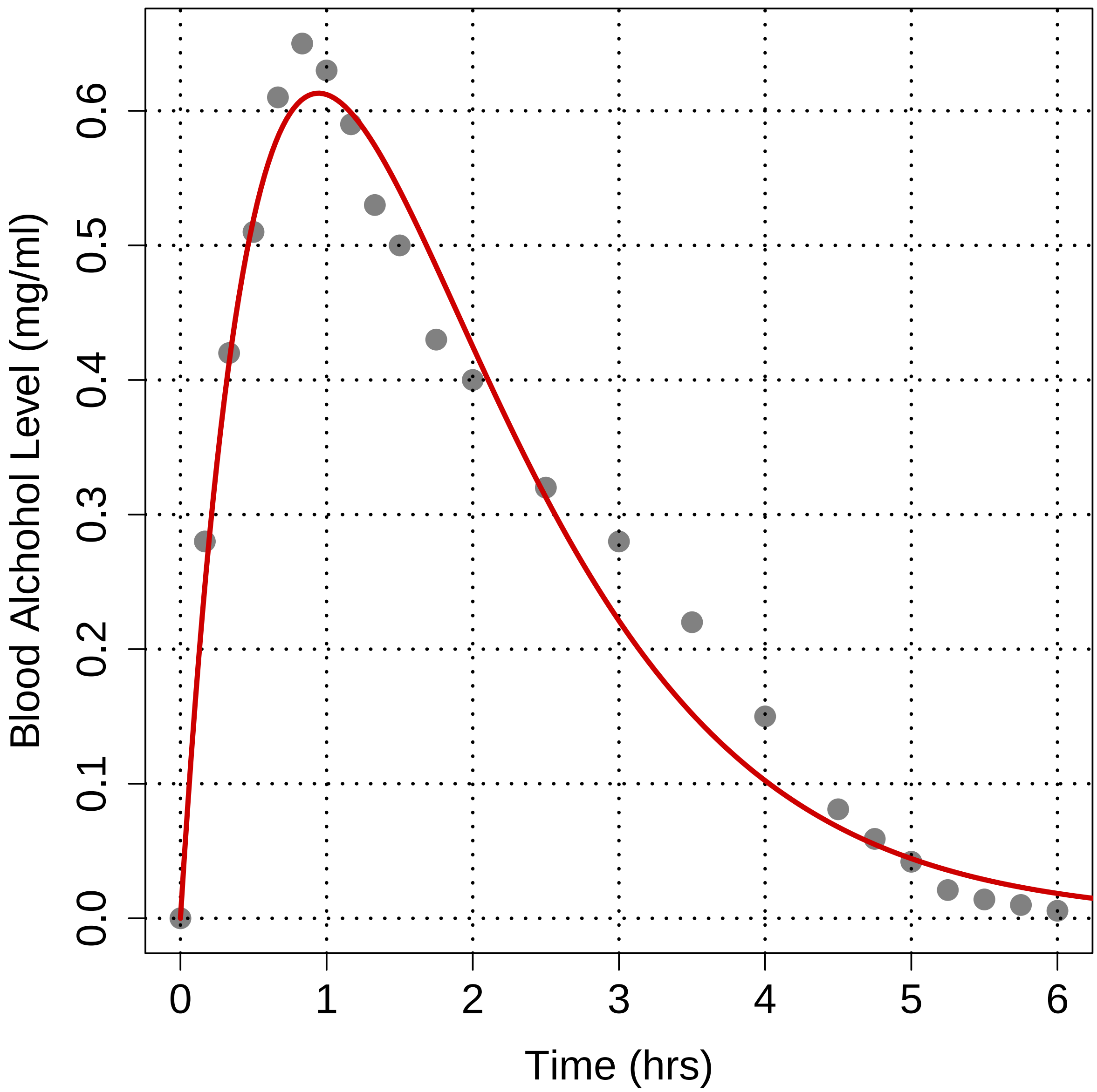

Figure 10: Blood alcohol concentration of eight fasting adult males after consuming a 95% ethanol oral dose of 45ml. [9] The curve is known as a surge function and given by $S(x) = 1.76393642046205xe^{-1.05841662684339x}$. Note that driving impairment begins around 0.5 mg/ml and the legal limit in most states is 0.8mg/ml. The dosage here is about 2.5 standard shots of vodka and about one and a half 16oz of 6% beer.

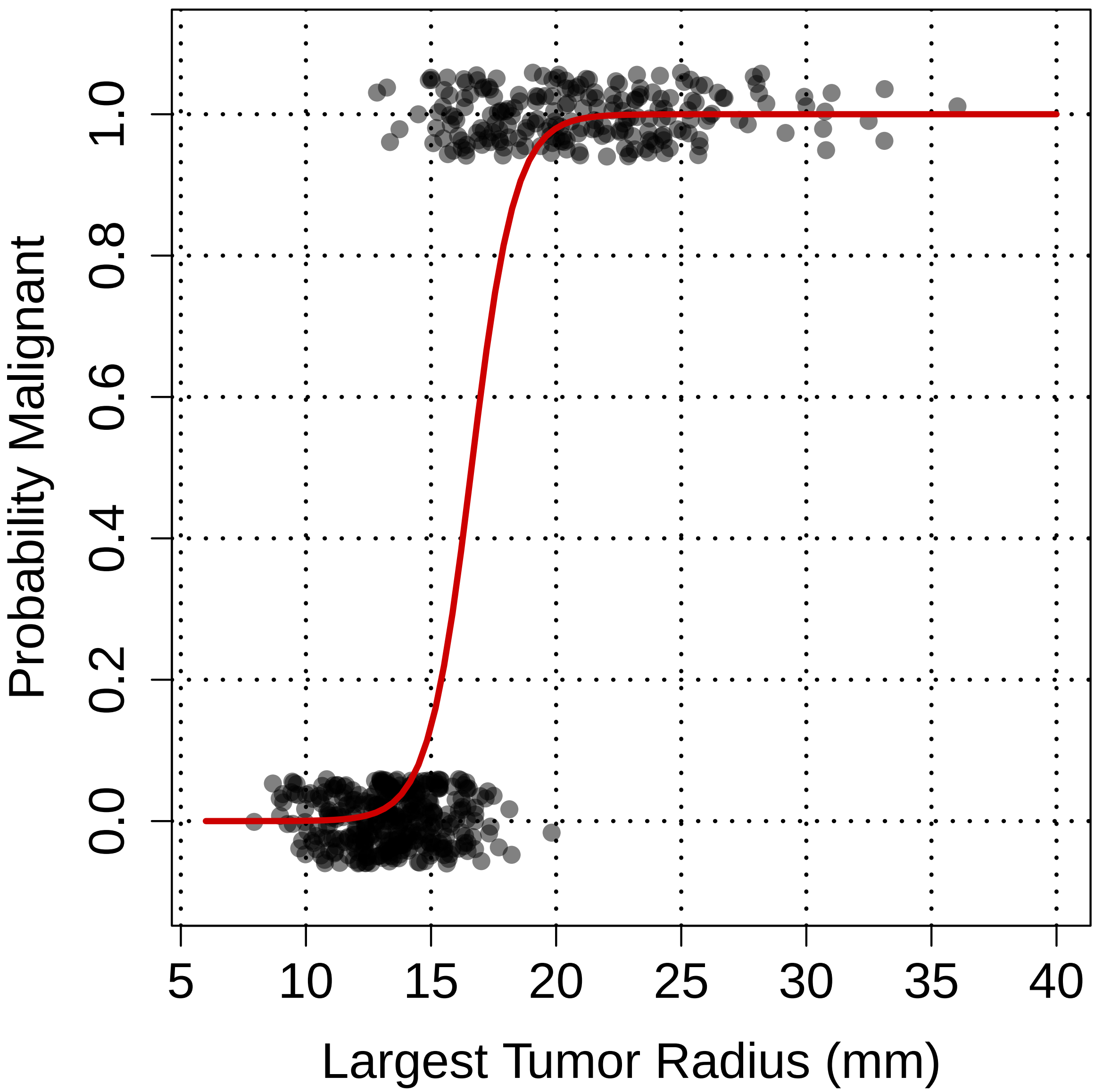

Figure 11: The data is from the Diagnostic Wisconsin Breast Cancer Data where "features are computed from a digitized image of a fine needle aspirate (FNA) of a breast mass. They describe characteristics of the cell nuclei present in the image." [10] The x–axis is the size of the largest radius of nuclei and the y–axis is the probability the breast mass is malignant. The curve is known as logistic regression and given by $L(x) = \dfrac{e^{-19.1744645062916+1.15383554750228x}}{1+e^{-19.1744645062916+1.15383554750228x}}$.

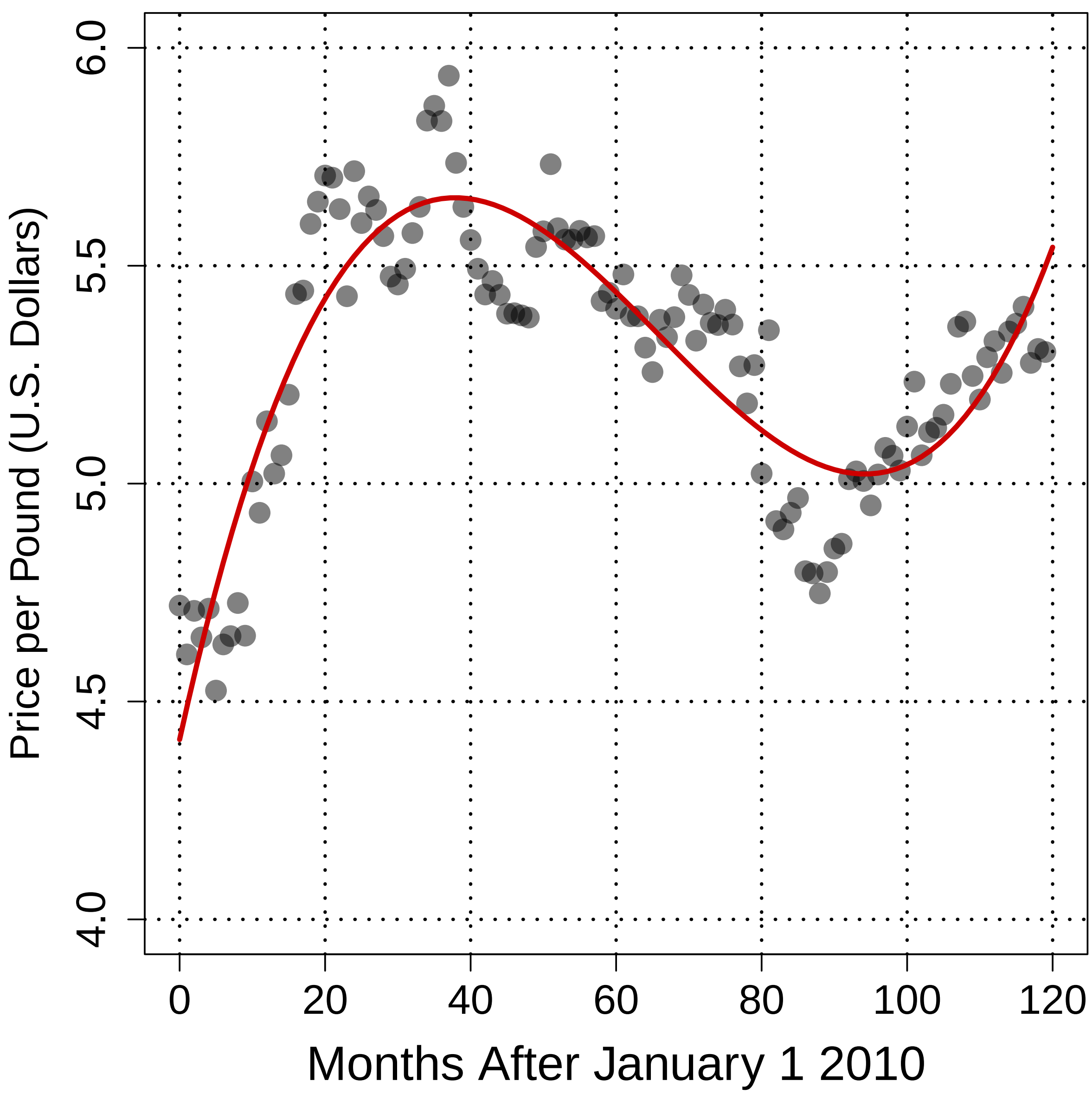

Figure 12: The average price of a pound of cheddar cheese in U.S. cities can be modeled by $Ch(t) = 4.41281093468948 + 0.0758375702747065t - 0.0014041565369063t^2 + 0.00000708849730870583t^3$ dollars, where t is the number of months since January 1, 2010 ($t = 0$). The data is from January 1, 2010 to December 1, 2019. [1]

1 Exercises

1. Use the $CO2(t)$ function and graph in figure 1 to answer the following questions.
   (a) What are the input and output units?
   (b) The data the model is based on begins and ends in what years?
   (c) Describe the graph using the definitions from Chapter **??**.
   (d) What is $CO2(63)$? Use your result in a sentence with proper context and units.
   (e) What is the yearly average CO2 in 1995? Use your result in a sentence with proper context and units.
   (f) According to the model when will CO2 reach 450 ppm? Use your result in a sentence with proper context and units.

2. Use the $Gtemp(t)$ function and graph in figure 2 to answer the following questions.
   (a) What are the input and output units?
   (b) The data the model is based on begins and ends in what years?
   (c) Describe the graph using the definitions from Chapter **??**.
   (d) What is $Gtemp(48)$? Use your result in a sentence with proper context and units.
   (e) What is the mean global temperature in 2020? Use your result in a sentence with proper context and units.
   (f) According to the model when will the mean global temperature reach 15.25 °C? Use your result in a sentence with proper context and units.

3. Use the $P(t)$ function and graph in figure 3 to answer the following questions.
   (a) What are the input and output units?
   (b) Describe the graph using the definitions from Chapter **??**.
   (c) What meaning does the inflection point have in the context of the data?
   (d) What was $P(11)$? Use your result in a sentence with proper context and units.
   (e) What was the number of people in extreme poverty in 2007? Use your result in a sentence with proper context and units.
   (f) According to the model in what year was the number of people in extreme poverty 900,000,000? Use your result in a sentence with proper context and units.

4. Use the $LeGdp(x)$ function and graph in figure 4 to answer the following questions.
   (a) What are the input and output units?
   (b) Should the model be used to extrapolate?
   (c) Describe the graph using the definitions from Chapter **??**.
   (d) What is $LeGdp(48)$? Use your result in a sentence with proper context and units.
   (e) What is the expected life expectancy for a country with a GDP per capita of $10,000? Use your result in a sentence with proper context and units.
   (f) According to the model what would be the GDP per capta of a country with a life expectancy of 85? Use your result in a sentence with proper context and units.

5. Use the $Wwind(t)$ function and graph in figure 5 to answer the following questions.
   (a) What are the input and output units?
   (b) The data the model is based on begins and ends in what years?
   (c) Describe the graph using the definitions from Chapter **??**.
   (d) What is $Wwind(37)$? Use your result in a sentence with proper context and units.
   (e) What is the cumulative installed world wind power in 2016? Use your result in a sentence with proper context and units.
   (f) According to the model when will the cumulative installed world wind power reach 1,000,000 MW? Use your result in a sentence with proper context and units.

6. Use the $Swind(t)$ function and graph in figure 6 to answer the following questions.
   (a) What are the input and output units?
   (b) The data the model is based on begins and ends in what years?
   (c) Describe the graph using the definitions from Chapter **??**.
   (d) What is $Swind(24)$? Use your result in a sentence with proper context and units.
   (e) What is the cumulative installed Spain wind power in 2019? Use your result in a sentence with proper context and units.
   (f) According to the model when will the cumulative installed Spain wind power reach 30,000 MW? Use your result in a sentence with proper context and units.

7. Use the $USwind(t)$ function and graph in figure 7 to answer the following questions.
   (a) What are the input and output units?
   (b) The data the model is based on begins and ends in what years?
   (c) Describe the graph using the definitions from Chapter **??**.
   (d) What is $USwind(33)$? Use your result in a sentence with proper context and units.
   (e) What is the cumulative installed U.S. wind power in 2017? Use your result in a sentence with proper context and units.
   (f) According to the model when did the cumulative installed U.S. wind power reach 85,000 MW? Use your result in a sentence with proper context and units.

8. Use the $ECus(x)$ and $ECw(x)$ functions and graph in figure 8 to answer the following questions.
   (a) What are the input and output units of both functions?
   (b) Describe both graphs using the definitions from Chapter **??**.
   (c) Which of the two graph has more concavity? Why?
   (d) What is $ECus(0.9)$ and $ECw(0.9)$? Use your results in a sentence or two with proper context and units. In your response compare the results and their meaning relative to each other.
   (e) What is the cumulative percent consumption for the U.S. and the world with a cumulative percent rank of 80%? Use your results in a sentence with proper context and units.
   (f) According to each model when does the cumulative percent consumption reach 75%? Use your results in a sentence with proper context and units.

9. Use the three Arctic sea ice extent curves in 9 to answer the following questions.
   (a) What are the input and output units of the functions?
   (b) Describe all three graphs using the definitions from Chapter ??.
   (c) Approximately when does the inflection point occur for each of the three years? Use your results in a sentence with proper context and units.
   (d) What is $AI\_1980(9)$ and $AI\_2012(9)$? Use your results in a sentence with proper context and units.
   (e) What is the Arctic sea ice extent for 2019 in July? Use your result in a sentence with proper context and units.
   (f) In 2012 when did Arctic sea ice extent reach 8 million square kilometers? Use your result in a sentence with proper context and units.
10. Use the $S(x)$ function and graph in 10 to answer the following questions.
   (a) What are the input and output units the function?
   (b) Describe the graph using the definitions from Chapter ??.
   (c) Approximately when does the inflection point occur? Use your result in a sentence with proper context and units.
   (d) What is $S(1.5)$? Use your result in a sentence with proper context and units.
   (e) What is the blood alcohol level two and half hours after the 95% ethnol oral does is consumed? Use your result in a sentence with proper context and units.
   (f) When is the blood alcohol level 0.35 mg/ml? Use your result in a sentence with proper context and units.
11. Use the $L(x)$ function and graph in 11 to answer the following questions.
   (a) What are the input and output units the function?
   (b) Describe the graph using the definitions from Chapter ??.
   (c) Approximately when does the inflection point occur? Use your result in a sentence with proper context and units.
   (d) What is $L(17)$? Use your result in a sentence with proper context and units.
   (e) What is the probability of a malignant tumor when the largest tumor radius is 18 mm? Use your result in a sentence with proper context and units.
   (f) If the probability a tumor is malignant is 70% then what is the size of the largest tumor? Use your result in a sentence with proper context and units.
12. Use the $Ch(t)$ function and graph in figure 12 to answer the following questions.
   (a) What are the input and output units?
   (b) The data the model is based on begins and ends at what dates?
   (c) Describe the graph using the definitions from Chapter ??.
   (d) What is $Ch(42)$? Use your result in a sentence with proper context and units.
   (e) What is the average price of a pound of cheddar cheese in U.S. cities on December 1, 2017? Use your result in a sentence with proper context and units.
   (f) According to the model when did the he average price of a pound of cheddar cheese in U.S. cities $5.00 per pound? Use your result in a sentence with proper context and units.

References

[1] Average Price: Cheddar Cheese, Natural (Cost per Pound/453.6 Grams) in U.S. City Average [APU0000710212]. U.S. Bureau of Labor Statistics. https://fred.stlouisfed.org/series/APU0000710212 (2022). Accessed 23 December 2022.

[2] Catalano, Michael T, Leise, Tanya L, and Pfaff, Thomas J. *Measuring Resource Inequality: The Gini Coefficient*. Numeracy 2(2): Article 4. 2009, 1–24. \href{http://services.bepress.com/numeracy/vol2/iss2/art4/}

[3] Hasell, J., Roser, M., Ortiz-Ospina, E., Arriagada, P., : Poverty. Our World in Data. https://ourworldindata.org/poverty (2022). Accessed 29 December 2022.

[4] *IEA Wind TCP Annual Reports*, IEA Wind, July 25, 2022, https://iea-wind.org/.

[5] National Snow and Ice Data Center. *Sea Ice Index*, July 25, 2022, https://nsidc.org/data/seaice_index/archives

[6] NOAA National Centers for Environmental information, Climate at a Glance: Global Time Series, published June 2022, retrieved on July 11, 2022 from https://www.ncei.noaa.gov/cag/

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[8] Tans, Pieter and Keeling, Ralph. *Trends in Atmospheric Carbon Dioxide*, NOAA, July 25, 2022, https://gml.noaa.gov/ccgg/trends/data.html.

[9] Wilkinson et. al., *Pharmacokinetics of Ethanol After Oral Administration in the Fasting State*. Journal of Pharmacokinetics and Biopharmaceutics 5(3):207-224, 1977, Accessed July 25, 2022, https://deepblue.lib.umich.edu/bitstream/handle/2027.42/45071/10928_2005_Article_BF01065396.pdf?sequence=1&isAllowed=y

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